AO 309 EXPERIMENTAL STRESS ANALYSIS

MODULE 1

SYLLABUS

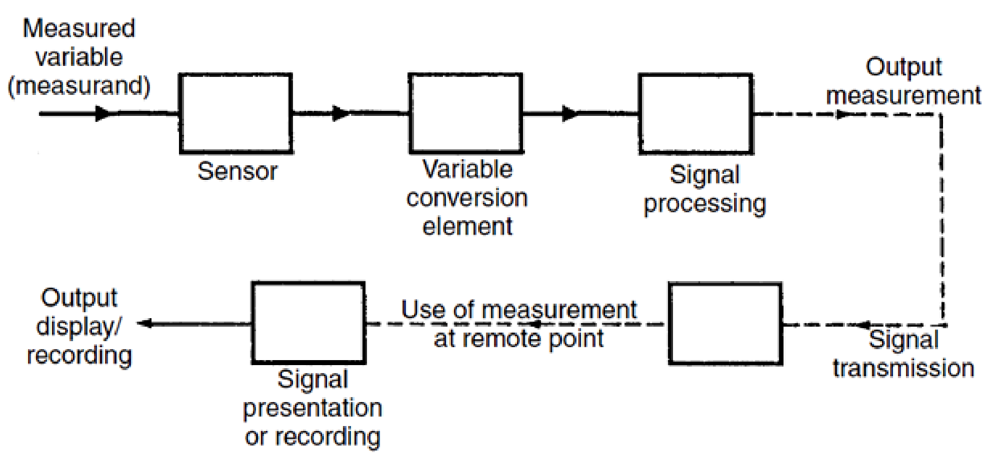
Principles of measurements, Accuracy, Sensitivity and range of measurements. Mechanical and Optical extensometers and their uses, Advantages and disadvantages. Acoustical and Electrical extensometers and their uses, Advantages and disadvantages. Capacitance gauges.

PRINCIPLES OF MEASUREMENTS

The massive growth in the application of computers to industrial process control and monitoring tasks has spawned a parallel growth in the requirement for instruments to measure, record and control process variables. As modern production techniques dictate working to tighter and tighter accuracy limits, and as economic forces limiting production costs become more severe, so the requirement for instruments to be both accurate and cheap becomes ever harder to satisfy. This latter problem is at the focal point of the research and development efforts of all instrument manufacturers.

Elements of a measurement system

A *measuring system* exists to provide information about the physical value of some variable being measured. In simple cases, the system can consist of only a single unit that gives an output reading or signal according to the magnitude of the unknown variable applied to it. However, in more complex measurement situations, a measuring system consists of several separate elements as shown in Figure 1.2. These components might be contained within one or more boxes, and the boxes holding individual measurement elements might be either close together or physically separate.



Elements of a measuring instrument

The first element in any measuring system is the primary *sensor*: this gives an output that is a function of the measurand (the input applied to it). For most but not all sensors, this function is at least approximately linear. Some examples of primary sensors are a liquid-in-glass thermometer, a thermocouple and a strain gauge. In the case of the mercury-in-glass thermometer, the output reading is given in terms of the level of the mercury, and so this particular primary sensor is also a complete measurement system in itself. However, in general, the primary sensor is only part of a measurement system.

Variable conversion elements are needed where the output variable of a primary transducer is in an inconvenient form and has to be converted to a more convenient form. For instance, the displacement-measuring strain gauge has an output in the form of a varying resistance. The resistance change cannot be easily measured and so it is converted to a change in voltage by a *bridge circuit*, which is a typical example of a variable conversion element. In some cases, the primary sensor and variable conversion element are combined, and the combination is known as a *transducer*.

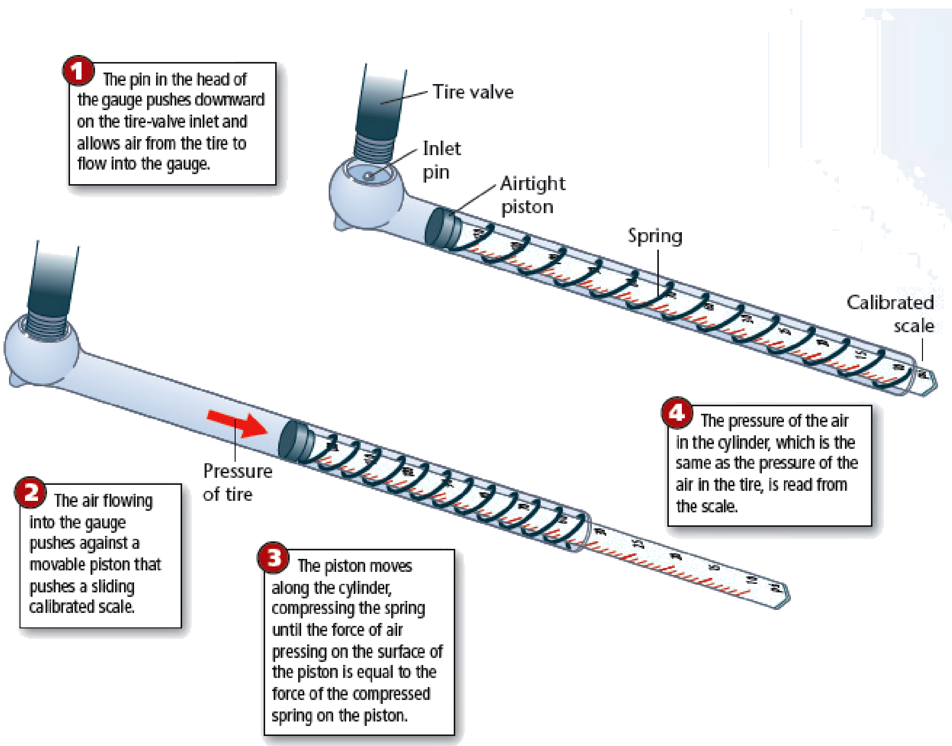
Signal processing elements exist to improve the quality of the output of a measurement system in some way. A very common type of signal processing element is the electronic amplifier, which amplifies the output of the primary transducer or variable conversion element, thus improving the sensitivity and resolution of measurement. This element of a measuring system is particularly important where the primary transducer has a low output. For example, thermocouples have a typical output of only a few millivolts. Other types of signal processing element are those that filter out induced noise and remove mean levels etc. In some devices, signal processing is incorporated into a transducer, which is then known as a *transmitter*.

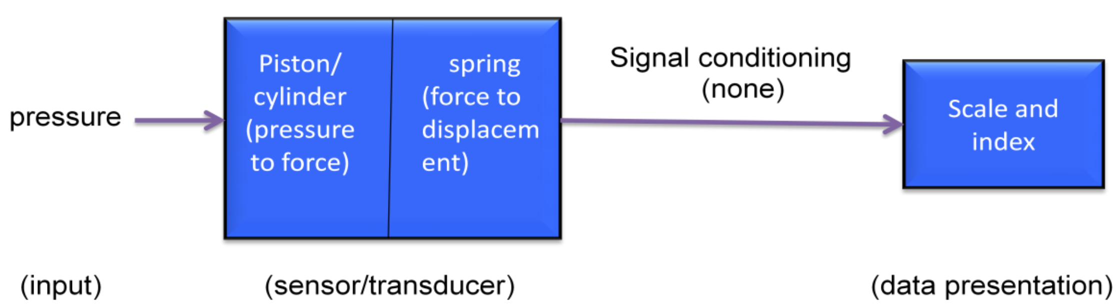
In addition to these three components just mentioned, some measurement systems have one or two other components, firstly to transmit the signal to some remote point and secondly to display or record the signal if it is not fed automatically into a feedback control system. Signal transmission is needed when the observation or application point of the output of a measurement system is some distance away from the site of the primary transducer.

The signal transmission element has traditionally consisted of single or multi-cored cable, which is often screened to minimize signal corruption by induced electrical noise. However, fibre-optic cables are being used in ever increasing numbers in modern installations, in part because of their low transmission loss and imperviousness to the effects of electrical and magnetic fields.

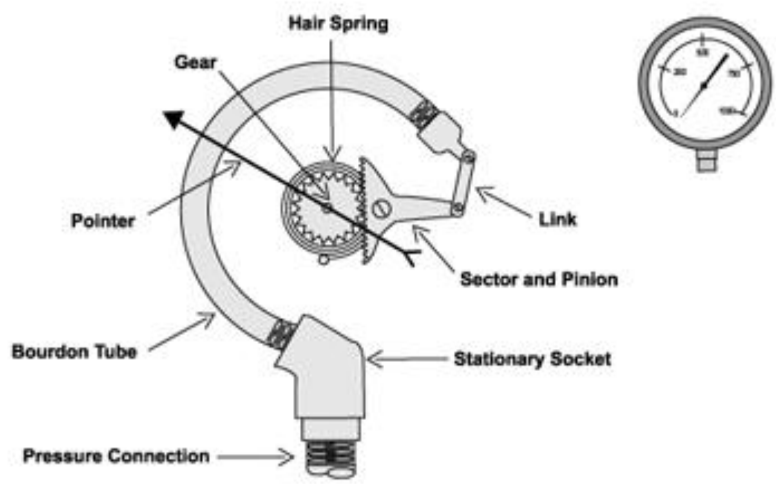
The final optional element in a measurement system is the point where the measured signal is utilized. In some cases, this element is omitted altogether because the measurement is used as part of an automatic control scheme, and the transmitted signal is fed directly into the control system. In other cases, this element in the measurement system takes the form either of a signal presentation unit or of a signal-recording unit.

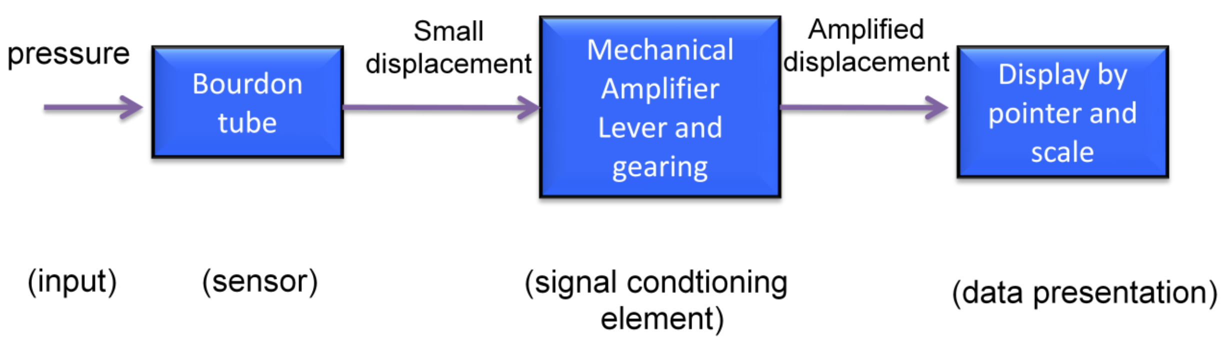
Example 1: Tire pressure gauge



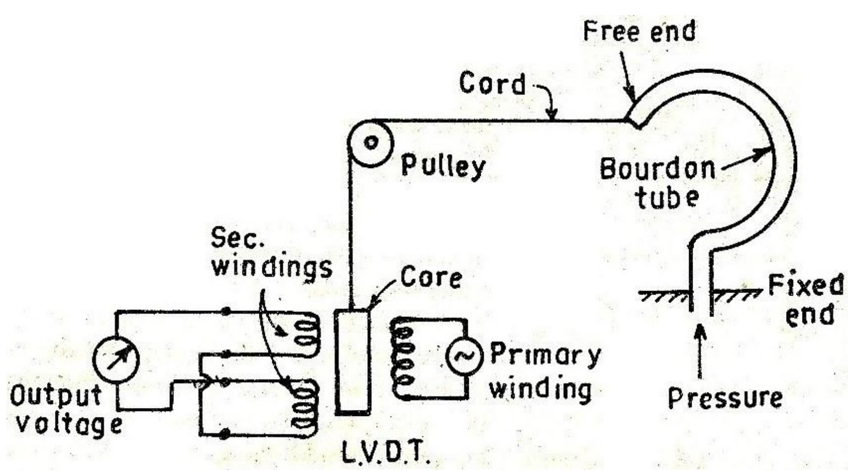


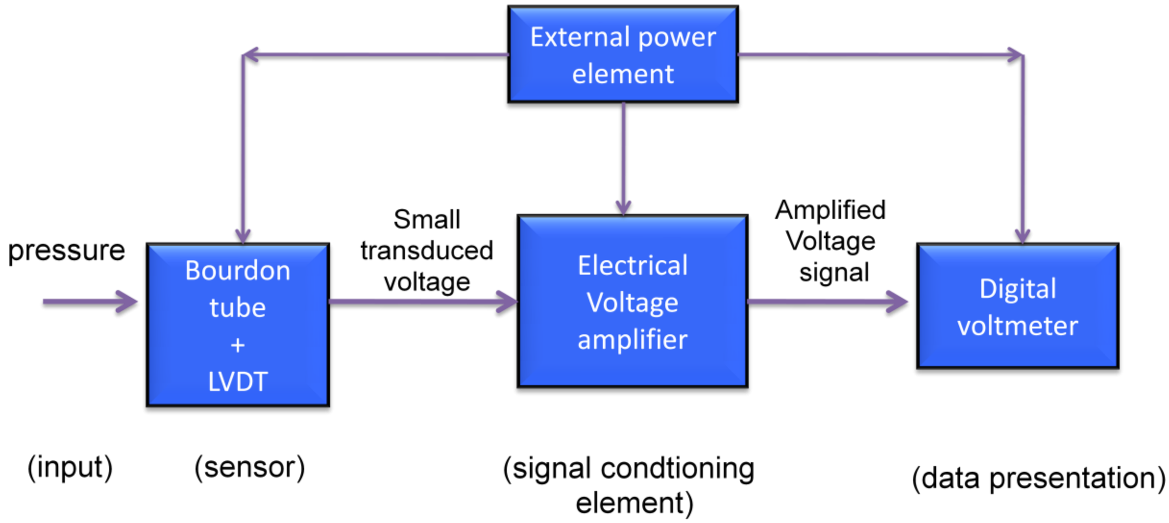
Example 2: Bourdon tube pressure gauge





Example 3: Bourdon tube pressure gauge with electrical read out





STATIC CHARACTERISTICS OF INSTRUMENTS

1. Accuracy and inaccuracy (measurement uncertainty)

The accuracy of an instrument is a measure of how close the output reading of the instrument is to the correct value. In practice, it is more usual to quote the inaccuracy figure rather than the accuracy figure for an instrument. Inaccuracy is the extent to which a reading might be wrong, and is often quoted as a percentage of the full-scale (f.s.) reading of an instrument. If, for example, a pressure gauge of range 0–10 bar has a quoted inaccuracy of ±1.0% f.s. (±1% of full-scale reading), then the maximum error to be expected in any reading is 0.1 bar. This means that when the instrument is reading 1.0 bar, the possible error is 10% of this value. For this reason, it is an important system design rule that instruments are chosen such that their range is appropriate to the spread of values being measured, in order that the best possible accuracy is maintained in instrument readings. Thus, if we were measuring pressures with expected values between 0 and 1 bar, we would not use an instrument with a range of 0–10 bar. The term measurement uncertainty is frequently used in place of inaccuracy.

2. Precision/repeatability/reproducibility

Precision is a term that describes an instrument's degree of freedom from random errors. If a large number of readings are taken of the same quantity by a high precision instrument, then the spread of readings will be very small. Precision is often, though incorrectly, confused with accuracy. High precision does not imply anything about measurement accuracy. A high precision instrument may have a low accuracy. Low accuracy measurements from a high precision instrument are normally caused by a bias in the measurements, which is removable by recalibration.

The terms repeatability and reproducibility mean approximately the same but are applied in different contexts as given below. Repeatability describes the closeness of output readings when the same input is applied repetitively over a short period of time, with the same measurement conditions, same instrument and observer, same location and same conditions of use maintained throughout. Reproducibility describes the closeness of output readings for the same input when there are changes in the method of measurement, observer, measuring instrument, location, conditions of use and time of measurement. Both terms thus describe the spread of output readings for the same input. This spread is referred to as repeatability if the measurement conditions are constant and as reproducibility if the measurement conditions vary.

3. Tolerance

Tolerance is a term that is closely related to accuracy and defines the maximum error that is to be expected in some value. Tolerance describes the maximum deviation of a manufactured component from some specified value.

4. Range or span

The range or span of an instrument defines the minimum and maximum values of a quantity that the instrument is designed to measure.

5. Sensitivity of measurement

The sensitivity of measurement is a measure of the change in instrument output that occurs when the quantity being measured changes by a given amount. Thus, sensitivity is the ratio:

$$\frac{\text{scale deflection}}{\text{value of measurand producing deflection}}$$

The sensitivity of measurement is therefore the slope of the straight line drawn on Figure 2.6. If, for example, a pressure of 2 bar produces a deflection of 10 degrees in a pressure transducer, the sensitivity of the instrument is 5 degrees/bar (assuming that the deflection is zero with zero pressure applied).

6. Threshold

If the input to an instrument is gradually increased from zero, the input will have to reach a certain minimum level before the change in the instrument output reading is of a large enough magnitude to be detectable. This minimum level of input is known as the threshold of the instrument. As an illustration, a car speedometer typically has a threshold of about 15 km/h. This means that, if the vehicle starts from rest and accelerates, no output reading is observed on the speedometer until the speed reaches 15 km/h.

7. Resolution

When an instrument is showing a particular output reading, there is a lower limit on the magnitude of the change in the input measured quantity that produces an observable change in the instrument output. Like threshold, resolution is sometimes specified as an absolute value and sometimes as a percentage of f.s. deflection. One of the major factors influencing the resolution of an instrument is how finely its output scale is divided into subdivisions. Using a car speedometer as an example again, this has subdivisions of typically 20 km/h. This means that when the needle is between the scale markings, we cannot estimate speed more accurately than to the nearest 5 km/h. This figure of 5 km/h thus represents the resolution of the instrument.

EXTENSOMETERS

Extensometer (strain gauge) is a device that is used to measure linear deformation. It is useful for stress-strain measurements. Its name comes from "extension-meter". Strain gauges are essentially devices that sense the change in length, magnify it and indicate it in some form.

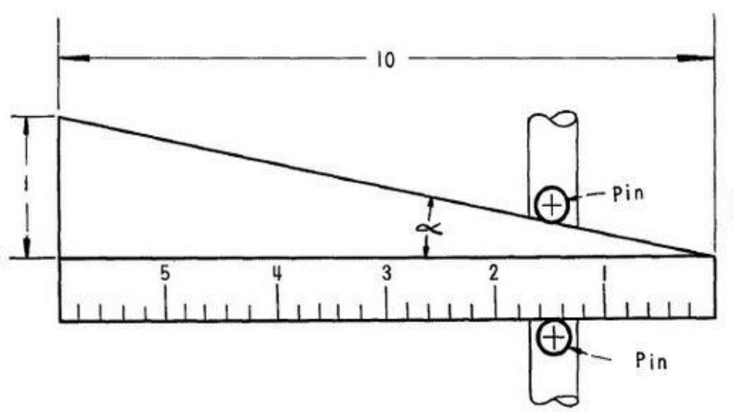
Gauge length:

An extensometer usually provided with two knife edges which clamped firmly in contact with the test component at a specific distance called gauge length. When the test component is strained, the two knife edges undergo a small relative displacement. This is amplified through a mechanical linkage and the magnified displacement or strain is displayed on a calibrated scale.

MECHANICAL EXTENSOMETERS

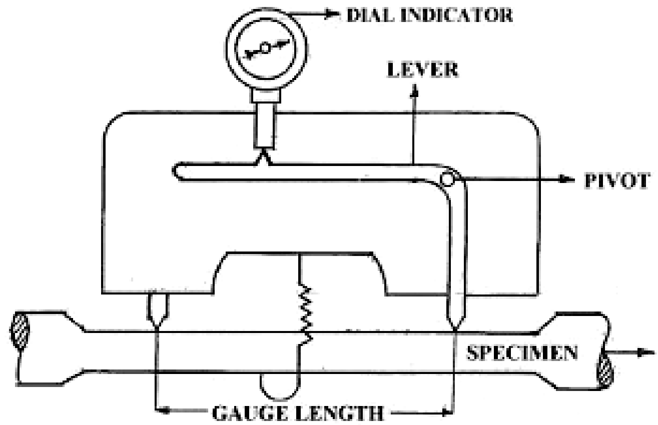
In mechanical extensometers, when the test component is strained, the two knife edges undergo a small relative displacement. This is amplified through a mechanical linkage and the magnified displacement or strain is displayed on a calibrated scale.

1. Wedge Gauge



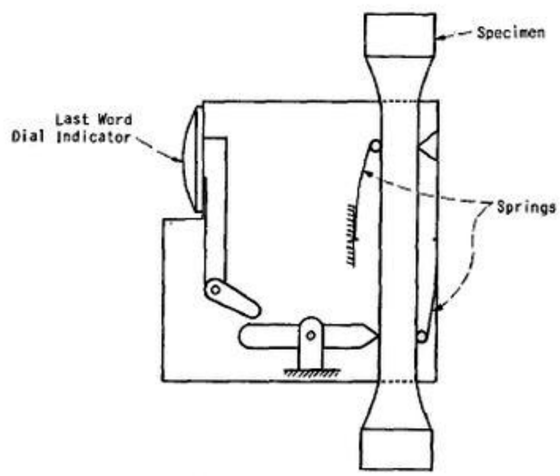
- This gauge is simply a triangular plate with a small included angle between two of the sides.
- When placed between two pins attached to the specimen, small values of pin separation distance can be measured.
- If the long sides are related by 1:10 slope, the magnification factor is 10.

2. **Berry Strain Gauge**



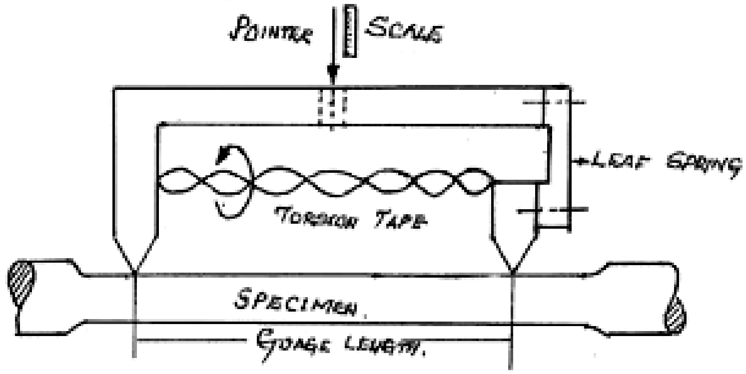
- It consists of a frame with two conically pointed contacts.
- It uses a system of lever and dial gauge to magnify the small displacement between the knife edges.
- It can measure strains down to 10 microstrain over a 50mm gauge length.

3. **Tinius Olsen Strain Gauge**



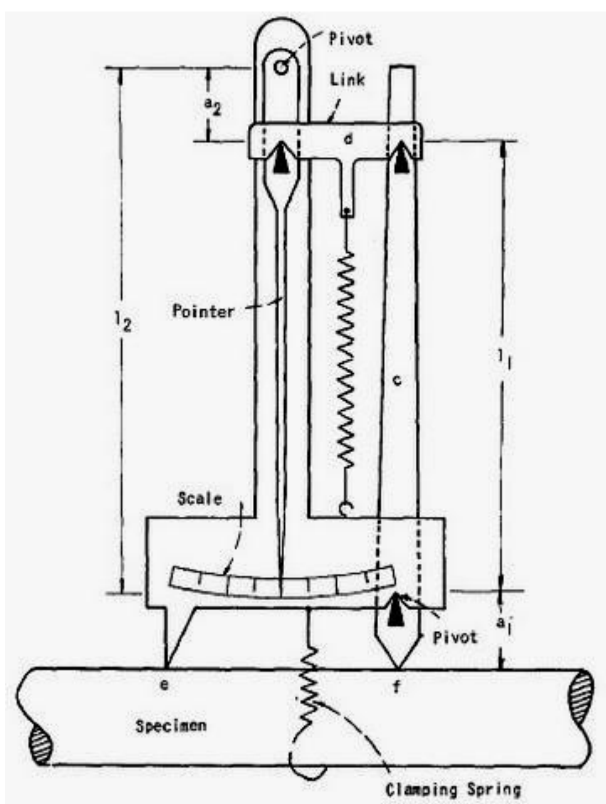
- It is very compact and convenient instrument for laboratory use.
- This combine lever and an indicator in which the strain in the 5cm gauge length may be read to the nearest 0.0025mm.
- The knife contacts acting on opposite sides of the specimen, give an average of strain values on the two sides with only one gauge reading.

4. **Johansson Extensometer**



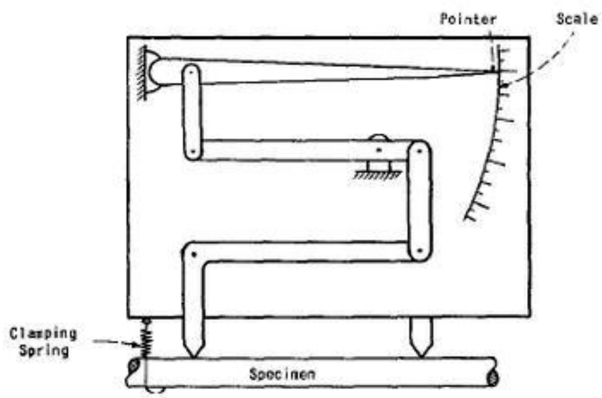
- The mechanical amplifying element is a twisted metal strip or torsion tape stretched between the knife edges.
- Half length of this strip is twisted in one direction while the other half is twisted in the opposite direction and a pointer is attached at the centre.
- The displacement of the knife edge i.e. stretching of the torsion tape is converted into a highly amplified rotational movement of the pointer.
- It can measure strain with a sensitivity of 5 microstrain over a 50mm gauge length.

5. **Huggenberger Extensometer**



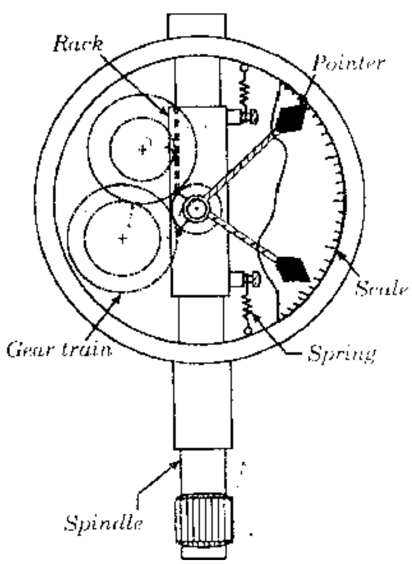
- In this a set of compound levers is used to magnify the displacement of the knife edges.
- It is highly accurate, reliable, light weight and self contained.
- The movable knife edge *f* rotates the lever *c* about the lover pivot.
- The lever *c* in turn rotates the pointer through the link *d*.
- The magnification ratio is given by $l_1 l_2 / a_1 a_2$
- Extensometers with this ratio varying between 300 and 2000 and with gauge lengths in the range 6.5 to 100mm are available.
- The sensitivity could be as high as 10 microstrain.

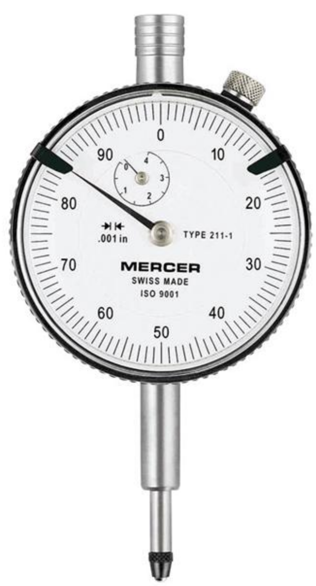
6. **Porter-Lipp Strain Gauge**



- The gauge length of this gauge is 25mm and magnification 300.

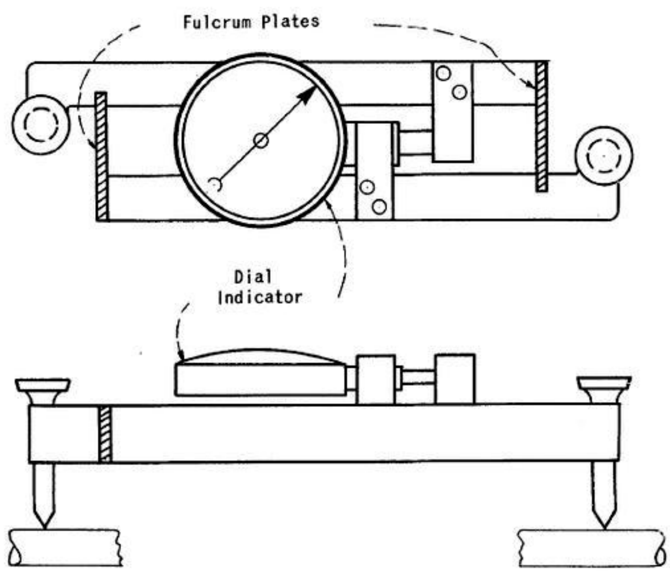
7. **Dial Gauge Indicator**





- The rack and pinion principle along with various types of gear trains is employed in gauges in which the magnification system types is incorporated in an indicating dial.
- In general a dial indicator consists of an encased gear train actuated by a rack cut in the spindle, which follows the motion to be measured.
- The gear train terminates with a light weight pointer which indicates spindle travel on a graduated dial.

8. **Whittemore Strain Gauge**



- Two bars are connected by flexible plates.
- A dial indicator is attached to one bar and the indicator spindle bears against the second bar.
- One gauge point is attached to each bar.
- For strain measurements the contact points are inserted into drilled holes defining a predetermined gauge length.
- The relative motion of the bars is indicated by a dial which reads in 0.0001inch divisions.

Advantages of mechanical strain gauge

- Ease with which they can be used
- Relatively low cost
- Require no special instrumentation

Disadvantages

- Relatively bulky size

- Long gage lengths and the variety of practical applications is extremely limited

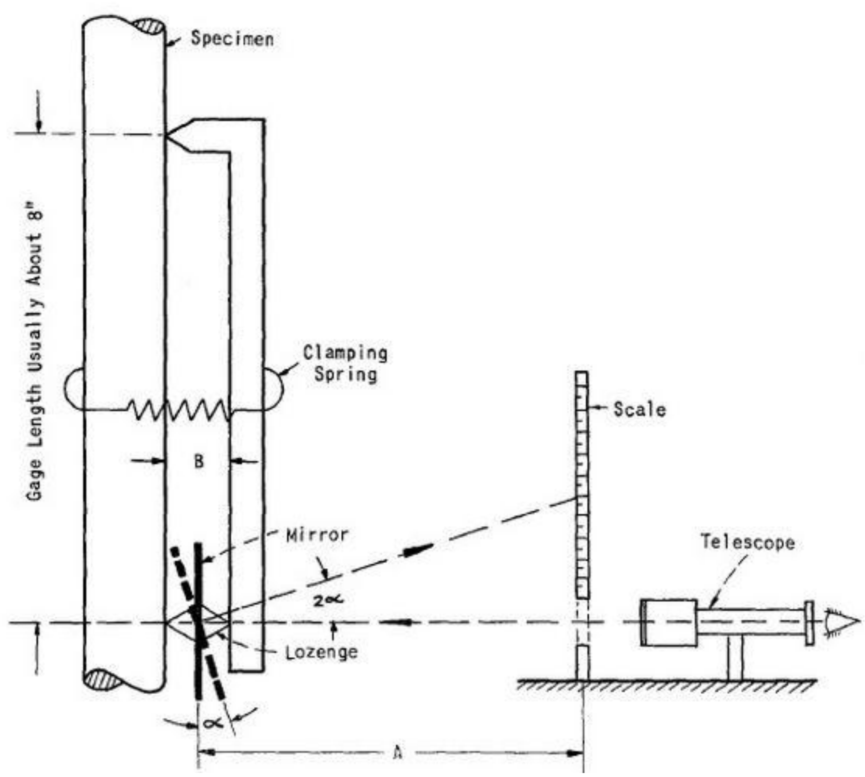
OPTICAL EXTENSOMETERS

A combination of mechanical and optical levers are used to amplify the relative displacement between the knife edges. The moving knife is pivoted so that it rotates while undergoing displacement.

Two types of optical extensometers are:

1. Marten's mirror extensometer
2. Tuckerman optical strain gauge

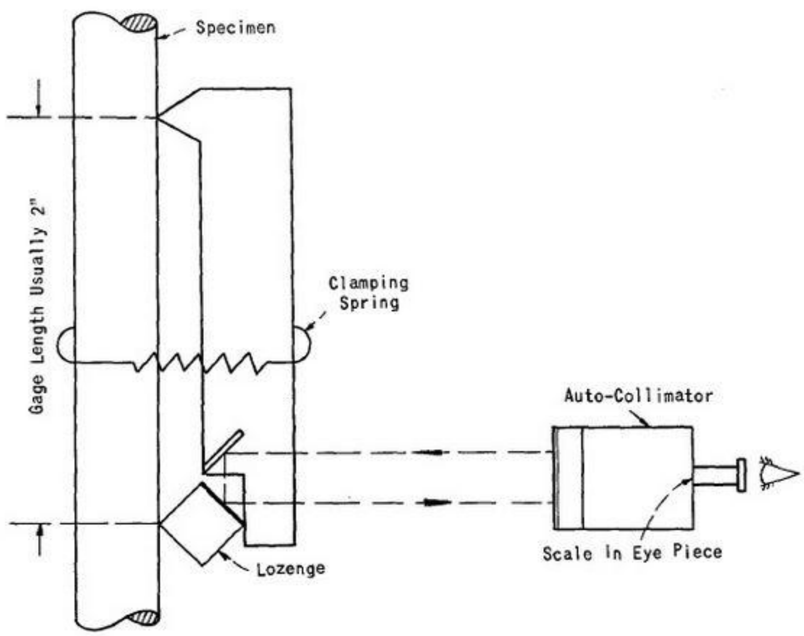
1. Marten's mirror extensometer



- The pivoting knife edge (lozenge) carries a mirror.
- The reflection of an illuminated scale in this mirror is viewed through the observing telescope.
- Any deformation of the structure to which this gauge is fixed, rotates the mirror and there by brings different portion of the scale into view.

- Thus the change in the reading on the scale is directly proportional to the deformation being measured.
- Magnification, $M = \frac{S}{\Delta L} \quad = \frac{A \tan 2\alpha}{B \sin \alpha} = \frac{2A}{B}$
- The distance B is usually 5mm with A 250 times B, giving a magnification of 500.

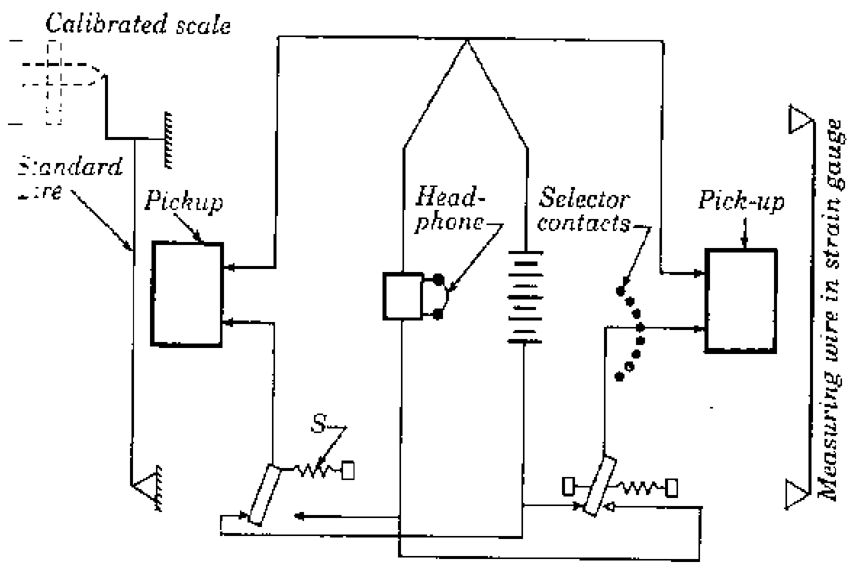
2. Tuckerman optical strain gauge



- Autocollimator carries both the source of a parallel beam of light and an optical system with reticle to measure the deflection of the reflected ray.
- A tungsten-carbide lozenge functions as the moving knife edge. One edge of this is polished to function as a mirror.
- The rotation of the lozenge resulting from a deformation of the structure deflects the incident parallel light beam back tom the measuring reticle.
- In this system any relative motion between specimen and the autocollimator will not affect the measurement.

- The sensitivity if this gauge is 2 microstrain.
- The gauge is available with a wide range of gauge lengths, starting from 6mm.

ACOUSTICAL EXTENSOMETERS



- The acoustical strain gauge consists essentially of a steel wire tensioned between two supports at predetermined distance apart.
- Variation of the distance alters the natural frequency of vibration of the wire and this change in frequency may be correlated with the change in strains causing it.
- An electro-magnet adjacent to the wire may be used to set the wire in vibration and this wire movement will then generate an oscillating electrical signal.
- The signal may be compared with the pitch of an adjustable standard wire, the degree of adjustment necessary to match the two signal frequencies being provided by a tensioning screw on the standard wire.
- Calibration of this screw allows a direct determination of the change of length of a measuring gauge to be made once the standard gauge has been tuned to match the frequency of the measuring wire.
- The fundamental frequency of a stretched wire may be estimated from the expression

$$f = \frac{1}{2L}\sqrt{\frac{P}{m}}$$
$$= \frac{1}{2L}\sqrt{\frac{\left(E\Delta L\right)/L}{m}A}$$

A = cross-sectional area of vibrating wire
E = Young's modulus of wire material
L = length of vibrating wire
m = mass per unit length of the wire
P = tensioning force in the wire
ΔL= increment in length of the vibrating wire.

ELECTRICAL EXTENSOMETERS

In an electrical strain gauge a change in length or strain produces a change in some electrical characteristics of the gauge. The greatest advantage common to all electrical gauges is the ease with which the electrical signal can be displayed, recorded or conditioned as required.

Three types of electrical gauges are in use:

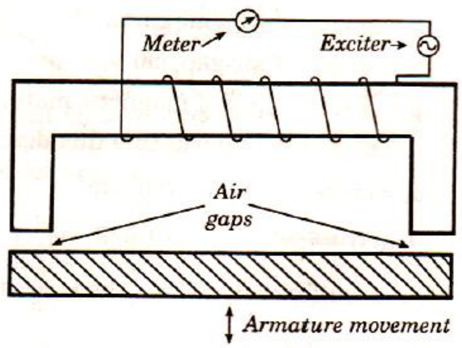
(i) Inductance gauges,
(ii) Capacitance gauges and
(iii) Electrical resistance gauges

Inductance Strain Gauges

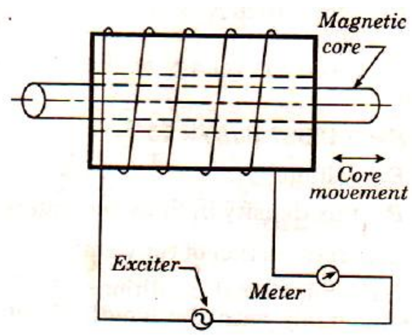
An electric inductance gauge is a device in which the mechanical quantity to be measured produces a change in the magnetic field, and, hence in the impedance, of a current-carrying coil. The impedance of a coil depends on its inductance and on its effective resistance and either or both of these quantities can be made sensitive to the mechanical quantity being measured.

$$Z = \sqrt{(2\pi fL)^2 + R^2}$$

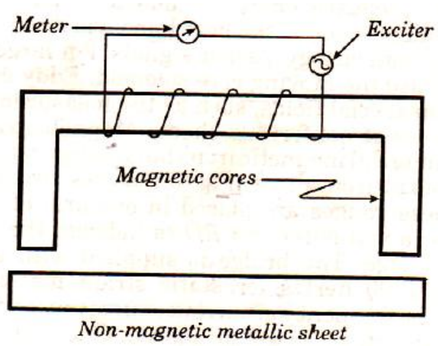
1. **Variable-air gap gauges**. In which the reluctance of the magnetic field is varied by changing the air gap**.**



2. **Movable-core solenoid gauges**. In which the reluctance of the magnetic circuit is varied by changing the position of the iron core in the coil.



3. **Eddy current gauges**. In which the losses in the magnetic circuit are varied by changing the thickness or position of the high-loss element inserted in the magnetic field.



Capacitance Strain Gauges

- The capacitance of a condenser can be varied by either varying the distance between the condenser plates or by varying the area.
- In a capacitance strain gauge the displacement resulting from the strain in the test component varies its capacitance.
- In the capacitance gauge shown in Fig. capacitance changes occur due to axial sliding of an outer cylinder relative to two concentric inner cylinders.

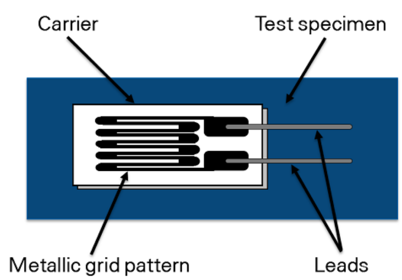
AO 309 EXPERIMENTAL STRESS ANALYSIS

MODULE 2

ELECTRICAL RESISTANCE STRAIN GAUGES

Electrical resistance strain gages are sensors fabricated from thin foil or wire-type conductors that respond to variations in their length with variations in their electrical resistance. These are used to measure linear strains that occur at surface points of an object when it responds to an actuating load. Strain gages are either bonded with adhesives to the surfaces of structures or are welded on. The measurement systems employ conventional elements, and have the important advantage of leading to electrical and treatable output. Circuitries commonly used are the potentiometer and the Wheatstone bridge circuit.

The metallic strain gauge consists of a very fine wire or, more commonly, metallic foil arranged in a grid pattern. The grid pattern maximizes the amount of metallic wire or foil subject to strain in the parallel direction.



Gauge construction

In the electrical resistance strain gauges the displacement or strain is measured as a function of the resistance change produced by the displacement in the gauging circuit.

An ideal strain gauge should have the following basic characteristics:

1. The gauge should be of extremely small size so as to adequately estimate strain at a point.
2. The gauge should be of significant mass to permit the recording of dynamic strains.
3. The gauge should b easy to attach to the member being analyzed and easy to handle.
4. The strain sensitivity and accuracy of the gauge should be sufficiently high.
5. The gauge should be unaffected by temperature, vibration, humidity or other ambient conditions.
6. The calibration constant for the gauge should be stable over a wide range of temperature and time.
7. The gauge should be capable of indicating both static and dynamic strains.
8. It should be possible to read the gauge either on location or remotely.

9. The gauge should exhibit linear response to strain.

10. The gauge and the associated equipment should be available at a reasonable cost.

11. The gauge should be suitable for use as a sensing element or other transducer.

Gage Factor

The resistance of a wire increases with increasing strain and decreases with decreasing strain. The resistance R of a uniform conductor with a length L, cross sectional area A, and specific resistance ρ is given by

$$R = \rho \frac{L}{A} = \frac{\rho L}{cD^2} \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots .. (1)$$

cD^2 is area of cross section of wire, A

Hence D is a sectional dimension and c is proportionality constant. For example, c=1 and π/4 for square and circular cross-section respectively.

Taking log on both side of eq (1)

$$\log R = \log \rho + \log L - \log c - 2\log D \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2)$$

When the wire is strained axially, each of the variables in eq (2) may change. Differentiate eq (2)

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - 2\frac{dD}{D} \dots \dots \dots \dots \dots \dots \dots \dots \dots . (a)$$

$$\frac{dR/R}{dL/L} = \frac{d\rho/\rho}{dL/L} + 1 - 2\frac{dD/D}{dL/L}$$

$$\frac{dR/R}{\epsilon} = \frac{d\rho/\rho}{\epsilon} + (1 + 2\vartheta)$$

$\in$ = strain in the wire

$\vartheta = \frac{\epsilon_2}{\epsilon_1} = -\frac{dD/D}{dL/L}$ = poisons ratio

This may be rewritten as

$$S_A = \frac{dR/R}{\epsilon} = (1 + 2\vartheta) + \frac{d\rho/\rho}{\epsilon} \quad \text{..................... (3)}$$

Where S_A is the **gauge factor** or sensitivity of the metallic alloy used in the conducer and is defined as the change in resistance per unit of initial resistance divided by the applied strain.

Eq (3) shows that the strain sensitivity of any alloy is due to two factors, namely the change in the dimension of the conductor, as expressed by the term (1+2υ) and change in specific resistance, as represented by (dρ/ρ)/ε. Experimental results show that S_A varies from about -12.1 to 6.1 for metallic alloys. Since most metal materials have the Poisson's ratio around 0.25 to 0.35, the (1+2υ), term in the strain sensitivity factor S_A is expected to be 1.5 to 1.7.

The value of the strain sensitivity S_A will depend upon the degree of cold-working imparted to the conductor in its formation, the impurities in the alloy, and the range of strain over which the measurement of S_A is made.

Many of the electrical resistance strain gauges produced today are fabricated from the copper-nickel alloy know as advance. Although the sensitivity factor S_A is usually provided by the strain gauge vendors,ngineers still need to choose the right gauge wire materials for their applications.

Material	Gage Factor (GF)	
	Low Strain	High Strain
Copper	2.6	2.2
Constantan*	2.1	1.9
Nickel	-12	2.7
Platinum	6.1	2.4
Silver	2.9	2.4
40% gold/palladium	0.9	1.9
Semiconductor**	~100	~600

* similar to “Ferry” and “Advance” and “Copel” alloys.
** semiconductor gage factors depend highly on the level and kind of doping used.

Table: A summary of the Gage Factor for different materials

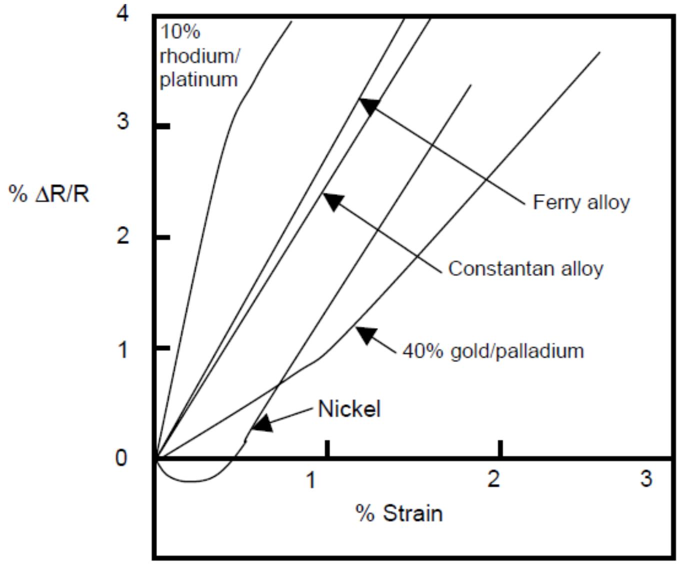
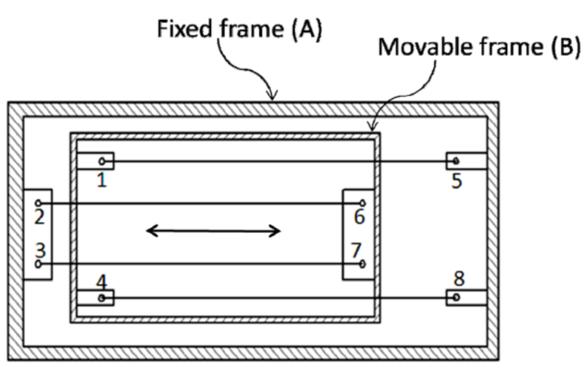


Fig: Change in Resistance with Strain for Various Strain Gage Materials

TYPES OF ELECTRICAL RESISTANCE STRAIN GAUGES

1. Unbonded gauges
2. Bonded gauges
3. Weldable gauges
4. Piezoresistive gauges

Unbonded Metallic Gauges

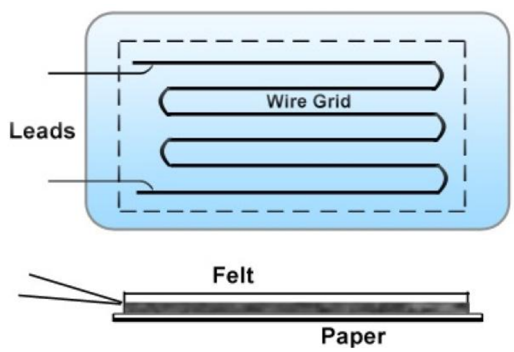


The basic mechanism of unbounded metallic stain gauge is illustrated in fig.

- Component parts A and B are both rigid frames; however, frames A is fixed and frame B is permitted to move in the direction indicated in the fig.

- In moving frame B, if the initial tension in wire 2-6 and 3-7 is increased, then the initial tension in wires 1-5 and 4-8 is reduced.
- These wires, which are fabricated from a strain sensitive material, represent four individual strain gauges.
- The movement of the frame B produces two positive changes in resistance and two negative changes.
- These four individual elements are connected in an appropriate fashion to a Wheatstone bridge circuit so that the resistance changes are added and the net output of the bridge is proportional to $4\Delta R/R$.
- Output voltages on the bridge are of the order of 40 mV full scale for an excitation voltage of 14 volts.

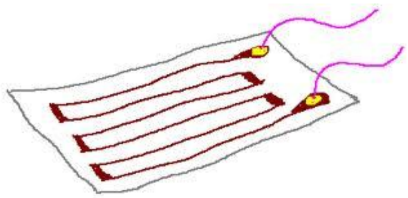
Bonded Metallic Gauges

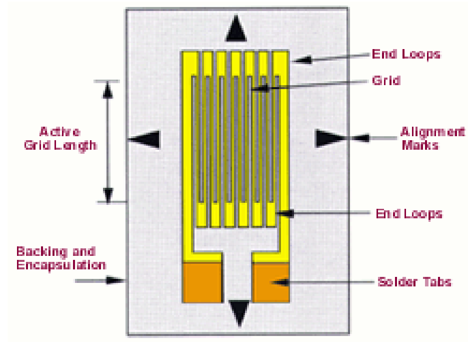


- The bonded metallic type of strain gauge consists of a strain sensitive conductor (wire) mounted on a small piece of paper or plastic backing.
- In us this gauge is cemented to the surface of the structural member to be tested.
- The wire grid may be flat type or wrap-around.
- In the flat type after attaching the lead wires to the ends of the grids, a second piece of paper is cemented over the wire as cover.
- In the wrap-around type, the wire is wound around a cylindrical core in the form of a close wound helix. This core is then flattened & cemented between layers of paper for the purpose of protection and insulation.
- Formerly only wrap-around gauges were available, but generally flat grid gauges are preferred as they are superior to wrap-around gauge in terms of hysteresis, creep, elevated temperature, performance, stability & current carrying capacity.

- The two layer of wire and three layer of paper result in a gauge which is approximately 0.006 inch thick. The backing material which carry the wire grid, protect it from damage.

Metal Foil Gauges

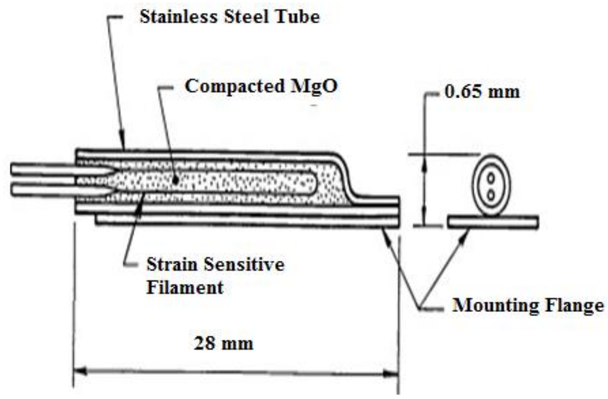




- The foil type of strain gauges has a foil grid made up of thin strain sensitive foil.
- The width of the foil is very large as compared to the thickness (microns) so that larger area of the gauge is for cementing.
- Currently the gauges are so thin (about 150μ inch) that they are often referred to as metal-film gauges. It is significant but proprietary developments have been made in highly refined photo-etching process used to form the grid configurations.
- The result of these manufacturing improvements is a gauge which has better tolerances on both the gauge factor and the initial resistance.
- The metal film or foil gauges are usually mounted on a thin epoxy carrier which is approximately 0.001 in. thick and extremely flexible.
- It is mounted on to a specimen with a suitable adhesive to give a strain measuring gauge in intimate contact with the surface of the specimen.

Weldable gauges

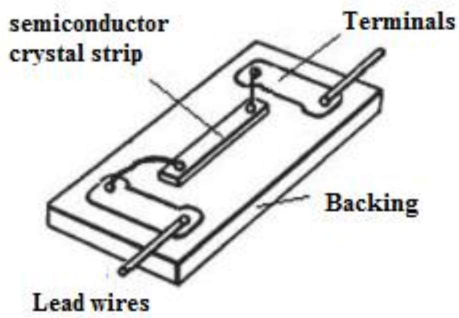
- Weldable strain gauges are easy to install in minutes in any environment compared to bonded type strain gauge.
- The weld able strain gauge consists of a strain sensitive element, the nickel Chromium or platinum Tungsten, housed within a small diameter stainless steel tube.
- The strain element is insulated from the tube with highly compacted ceramic insulation.
- This gauge is subsequently spot welded to structure under test and provides bonding to transfer the strain.



- The test specimen which is put into tension or compression, the stress is transmitted through the weld to mounting flange and in to strain tube. These gauges can be used for static or dynamic applications.
- The wire inside the case is reduced in diameter by an etching process; hence the minimum gauge resistance is obtained with very small-diameter high resistance wire rather than by forming a grid of larger-diameter wire.
- At present these weld able gauges are available with resistance ranging from 60 to 240 Ω and they are suitable for use in the range from -390 to 750 ^{0}F.

Piezoresistive Gauges

- Piezoresistive strain gauges are a semiconductor material which changes in resistance when the material stretched or compressed.
- Typically they are made from N or P type silicon and are either manufactured as separate elements for bonding to the surface of a sensing diaphragm.
- Filaments for semiconductor strain gauges are cut in rectangular cross section with cross sectional dimensions 0.127x0.0127mm. The gauge lengths vary from 0.25 to 13mm.
- Epoxies and phenolics are used as backing materials. The operating temperature range us -268 ^{0}C to +260 ^{0}C
- The main benefit of piezoresistive strain gages is that they have a much higher gauge factor than bonded foil strain gauges, which results in a significantly higher sensitivity and less noisy output signal.



MATERIALS USED FOR ELECTRICAL RESISTANCE STRAIN GAUGES

Grid Materials

The desirable features in a grid material are :

- High gauge factor
- High resistance
- Low temperature sensitivity
- High electrical stability
- High endurance limit
- Good solderability and workability
- Low hysteresis
- Good corrosion resistance
- Low thermal e.m.f. when joined with other metals

The important alloys or its equivalent that are commonly used as grid materials are:

(i) Constantan or Advance
(ii) Annealed Constantan
(iii) Iso-elastic
(iv) Nichrome V
(v) Karma (Ni-Cr-Al alloy with iron)
(vi) Iron-chromium aluminium alloy (Armor D)
(vii) Platinum-tungsten alloys.

The most widely used alloy for strain-gauge grids is a copper-nickel alloy known under such trade names as Constantan, Advance, etc.

Material	Composition (%)	Gauge Factor
Constantan (Advance)	Cu=55, Ni=45	2.1
Iso-elastic	Ni=36,Cr=8,Fe=52,Mo=4	3.5
Karma	Ni=75,Fe=12,Cr=11,Mn=2	2.5
Nichrome V	Ni=80, Cr=20	2.1

i) Constantan

- Constantan alloy has a reasonably high strain sensitivity or gauge factor, which is relatively insensitive to strain level and temperature.
- Its specific resistance is sufficiently high as to yield desired resistance values in even very small gauge lengths.
- Further, it has a good fatigue limit and a relatively high elongation capability. However at temperatures above about 65°C Constantan generally exhibits a continuous zero drift which is a serious drawback when measurements have to be taken over a period of hours or days.

ii) Annealed Constantan

- When very large strain have to be measured, annealed Constantan is usually used as grid material.
- Annealed Constantan in gauge lengths of 3 mm and larger can be strained to 20%.
- On account of the tendency for premature grid failure under repeated straining and zero drift, annealed Constantan is not suitable for cyclic-strain applications.
- Also, annealed Constantan is not amenable for self-temperature compensation.

iii) Iso-elastic

- Isoelastic alloy has a high gauge factor of 3.5.
- It has also better fatigue properties than Constantan.
- As its temperature coefficient of resistance is rather high, it is only suited for dynamic strain measurements.
- This alloy is usually suitable for strain measurements within ±5000 μm/m.

iv) Karma alloy

- Karma alloy has good fatigue life and excellent zero stability for temperature up to 300°C.
- It is best suited for high-precision static-strain measurements over an extended period of time (months or years) or for lesser periods at elevated temperatures (-269° to +290°C).
- Karma alloy can be processed for self-temperature compensation and used on test materials with different thermal expansion coefficients.

v) Nichrome V

- Owing to its high temperature sensitivity nickel-chromium alloys such as Nichrome V are used for dynamic strain measurements.
- This alloy is usually not available in the self-temperature-compensated form.
- However, with suitable temperature compensation, it may be used for static-strain measurements *to* 650°C and dynamic strain measurements to 1000°C.

Iron-chromium-aluminium alloys and platinum-tungsten alloys are suitable for operation at elevated temperatures. Platinum-tungsten alloys have high stability and fatigue life at elevated temperatures. It is recommended for static-strain measurements to 650°C and dynamic-strain measurements to 800°C.

Backing Materials

- A strain-gauge grid is normally supported on some form of carrier material.
- This backing material transmits the strain from the test material to the grid and provides electrical insulation between the grid and the test material, dimensional stability, and also provides some degree of mechanical protection for the delicate sensing element.
- A good carrier material should have the following desirable characteristics :
  - ✓ Minimum thickness
  - ✓ High mechanical strength
  - ✓ High dielectric strength
  - ✓ Non-hygroscopic
  - ✓ Minimum temperature restrictions
  - ✓ Good adherence to cement used

i) Paper

- Paper as a backing material is used mostly with wire strain gauges.

- When the gauge with paper backing is bonded to the test component with a nitrocellulose cement, the adhesive penetrates and fills the pores in the paper and ensures satisfactory bonding.
- This installation can be used for measurement of strains up to 100,000 μm/m.
- As paper absorbs moisture easily, paper backed gauges bonded with nitrocellulose cement are not suitable for long periods or field measurements.
- The operating temperature range of paper-backed gauges is -50°C to +80°C.

ii) Polyimide

- Polyimide as backing material is highly attractive as it is tough and highly flexible; it can be easily contoured to fit very small radii.
- Polyimide backing is used mostly for foil gauges.
- Polyimide-backed gauges are less sensitive to mechanical damage during installation and are widely used for general purpose static and dynamic stress analysis.
- Typical operating temperature range for foil gauges with this backing is -195° to +175°C.

iii) Epoxy plastics

- Epoxy plastics are suitable as backing material for both wire and foil gauges.
- Epoxy backing is more frequently used for foil gauges.
- The typical operating temperature range for epoxy backed gauges is -160° to +120°C.

iv) Epoxy-phenolic resin

- Epoxy-phenolic resin as a backing material is attractive as it gives a thin, flexible and easy to bond backing with low creep.
- Epoxy-phenolic-backed foil *gauges* are well-suited for field applications and for use in transducers.
- Wire gauges with epoxy-phenolic backing are also available.
- The operating temperature range for these gauges is -50° to +180°C.

v) Glass-fibre-reinforced epoxy-phenolic

- Glass-fibre-reinforced epoxy-phenolic backing material is a good choice for static and dynamic strain measurements over a wide temperature range of -269°C to +290°C.
- In short-term applications it can be used up to a maximum temperature of 400°C.
- However, the maximum elongation of this material is limited to about 20,000μm/m.

vi) Phenolic

- Strain gauges with phenolic (Bakelite) backing are well-known for their long-term stability and resistance to creep.
- They can be used at temperatures up to about 200°C.
- Gauges with glass-reinforced phenolic can be used
  for static and dynamic strain measurements over a wider temperature of -160°C to 200°C .

vii) Strippable backing

- Open-faced gauges with strippable backing are available for use at very high or at cryogenic temperatures.
- The grid element in these gauges is stuck to the test component using a suitable ceramic adhesive and the backing is then peeled off.
- Typical materials for the strippable backing are vinyl and glass-reinforced teflon.
- These gauges can be used typically up to about 900°C.

<u>Adhesives</u>

- The bondable strain gauges are attached to the test specimen by some form of cement or adhesive.
- The strain gauge adhesive should be sufficiently elastic to faithfully transfer strain in the test component to the gauge- sensing element or grid.
- For optimum performance, the adhesive prescribed by the gauge manufacturer should be used and the recommended procedure for cementing the gauge should be strictly followed.
- The following are the desirable characteristics of the bonding cement:
  - ✓ High mechanical strength
  - ✓ High creep resistance
  - ✓ High dielectric strength
  - ✓ Minimum temperature restrictions
  - ✓ Good adherence
  - ✓ Minimum moisture absorption
  - ✓ Ease of application
  - ✓ Low setting time

i) Nitro-Cellulose Cement

- Nitro-cellulose cement is commonly used to mount paper-backed gauges.
- Since these cements contain a very large fraction of solvent (about 85%), the bonded gauge should be cured to remove all the solvents by evaporation.

- The curing time varies with the percentage of solvent, relative humidity, curing temperature and also the purpose for which the gauge is used.
- For short-time tests using thin paper gauges, curing at room temperature for several hours may suffice.
- The curing time for wrap-around gauges is 5 to 10 times longer than for flat-grid gauges.
- As nitro-cellulose cements are hygroscopic, the cured gauge should be immediately protected by a moisture resistance coating.
- This will ensure electrical and dimensional stability of the gauge installation.

ii) Epoxy Cements

- Two types of epoxy cements -room temperature epoxies and thermosetting epoxies -are commonly used.
- Both types have two constituents — a monomer and a hardening agent. Mixing of a monomer with the hardening agent induces polymerization.
- Room-temperature epoxies use amine-type hardening agent while thermosetting epoxies require anhydride type of hardening agent.
- Thermosetting epoxies need a curing temperature in excess of 120°C for several hours for complete polymerization.

iii) Cyanoacrylate cement

- Cyaneacrylate cement cures rapidly at room temperature
- It is compatible with most of the test materials and strain gauge backing materials. A firm thumb pressure for about a minute is sufficient to induce polymerization at room temperature.
- A strain gauge bonded with this adhesive can be used approximately 10 min after bonding. Its performance deteriorates with time, elevated temperature and moisture absorption.
- When protected with coatings like microcrystalline wax or silicone rubber, the life of the strain gauge bonded with this adhesive can be extended to 1 or 2 years.
- The typical -operating temperature range for this adhesive is -75° to + 65°C.

iv) Phenolic Adhesives

- Phenolics are single component thermosetting adhesives requiring curing at an elevated temperature of 120° to 175°C for a fairly long period.
- In most applications these adhesives have been replaced by epoxy-phenolic adhesives.

Strain Gauge Lead Wires and Connections

(a) Lead Wires

- Lead wires are needed to transmit the electrical signals from strain gauges to the strain measuring instrument.
- The lead-wire system connected to the gauges must perform satisfactorily under all environmental conditions.
- The lead wires should have low resistance and low temperature coefficient of resistance.
- They should not introduce significant resistance change, or generate or transmit electrical noise.
- As copper has low specific resistance, it is commonly used as material for lead wires.
- However, copper has a large temperature coefficient of resistance and has poor corrosion and fatigue resistance. Tinned, plated or metal clad solid copper wires have superior corrosion resistance.
- Stranded copper wires are flexible. Hence they are used when relative motion between the lead wire and the component is expected.
- Nickel-chromium alloy lead wires are prescribed in high-temperature applications.
- These wires are suitable for temperatures up to about 370°C. As it has a high specific resistance, only short-length lead wires should be used.

(b) Insulation

- The lead wires are always covered with an insulating material.
- Some of the insulating materials used for this purpose are:
    a) Teflon (-269^{0}C to+260^{0}C)
    b) polyimide (-269^{o}C to +315°C)
    c) vinyl (-50° to + 80°C)
    d) polyurethane (- 75° to +150°C)
    e) fibreglass (-269° to +480°C)

(c) Connections

- Lead wires are usually soldered to the strain gauge leads.
- The melting point of the solder chosen should he at least 15^{0}C higher than the test temperature.

Protective Coatings

- Strain gauge installation requires various degrees of protection to avoid gauge instability and mechanical damage.
- Gauges are easily degraded by any chemical attack. Moisture, fingerprints flux residues etc.
- Open faced gauges should always have a suitable coating applied as soon possible after installation.
- Factors influencing the selection of a coating are the test environment, test duration and accuracy requirements.
- Coatings used for protecting the gauge installations should not appreciably stiffer than the specimen**.**

- **Microcrystalline wax** is one of the first coating material used in gauge work. The wax has to be melted and the specimen preheated before application. A wax coating is a most effective barrier to water or humidity but provides very little mechanical protection. Wax coating are generally limited to use within a temperature range of -73 to 65°C.
- **Plastic coatings such as liquid vinyls or flexible spray resins** systems are resistant to many solvents, chemicals, and in some degree, oil and grease. Vinyl coatings are particularly useful where operation is in the cryogenic temperature range. Epoxy coatings have a higher operating temperature limit than vinyls.
- **Silicon base materials** provide good protection against chemicals and moisture and can be used at temperature upto 425°C.
- **Synthetic coatings, including neoprene, polysulphides, butyl polymers** and room temperature vulcanizing type silicon rubber offer excellent mechanical and environmental protection against moisture, fresh and salt water immersion and spray. The operating temperature range of these materials varies between -73° to + 230.

CROSS SENSITIVITY

Cross sensitivity or Transverse sensitivity in a strain gage refers to the behavior of the gage in responding to strains which are perpendicular to the primary sensing axis of the gage. Ideally, it would be preferable if strain gages were completely insensitive to transverse strains. In practice, most gages exhibit some degree of transverse sensitivity; but the effect is ordinarily quite small, and of the order of several percent of the axial sensitivity.

The transverse sensitivity of these gages is due almost entirely to the fact that a portion of the wire in the end loop lies in the transverse direction. The end loops in the foil gauges have a relatively large cross section, hence their contribution to transverse sensitivity is quite small.

Errors Due to Transverse Sensitivity

In the particular case of uniaxial stress in a material with a Poisson's ratio of 0.285, the error is zero because the gage factor given by the manufacturer was measured in such a uniaxial stress field and already includes the effect of the Poisson strain. It is important to note that when a strain gage is used under any conditions other than those employed in the gage-factor calibration, there is always some degree of error due to transverse sensitivity.

In general, then, a strain gage actually has two gage factors, *Fa* and *Ft*, which refer to the gage factors as determined in a uniaxial strain field (not uniaxial stress) with, respectively, the gage axes aligned parallel to and perpendicular to the strain field. For any strain field, the output of the strain gage can be expressed as:

$$\frac{\Delta R}{R} = F_a \varepsilon_a + F_t \varepsilon_t \qquad (1)$$

where: $\varepsilon_a, \varepsilon_t$ = strains parallel to and perpendicular to the gage axis, or the gridlines in the gage.

F_a = axial gage factor.

F_t = transverse gage factor.

Or,

$$\frac{\Delta R}{R} = F_a(\varepsilon_a + K_t \varepsilon_t) \qquad (2)$$

where: $K_t = \frac{F_t}{F_a}$ = transverse sensitivity coefficient, referred to from here on as the "transverse sensitivity".

When the gage is calibrated for gage factor in a uniaxial stress field on a material with Poisson's ratio, ν_0,

$$\varepsilon_t = -\nu_0 \varepsilon_a$$

Therefore,

$$\frac{\Delta R}{R} = F_a\left(\varepsilon_a - K_t \nu_0 \varepsilon_a\right)$$

or,

$$\frac{\Delta R}{R} = F_a\left(1 - \nu_0 K_t\right)\varepsilon_a \qquad (3)$$

The strain gage manufacturers commonly write this as:

$$\frac{\Delta R}{R} = F\varepsilon \qquad (3a)$$

where: F = manufacturer's gage factor, which is deceptively simple in appearance, since, in reality:

$$F = F_a\left(1 - \nu_0 K_t\right) \qquad (4)$$

Furthermore, ε is actually ε_a, the strain along the gage axis (and only one of two strains sensed by the gage during calibration) when the gage is aligned with the maximum principal stress axis in a uniaxial stress (not uniaxial strain) field, on a material with υ_0= 0.285. Errors and confusion occur through failure to fully comprehend and always account for the real meanings of F and ε as used by the manufacturers.

It is imperative to realize that for any strain field except that corresponding to a uniaxial stress field (and even in the latter case, with the gage mounted along any direction except the maximum principal stress axis, or on any material with Poisson's ratio other than 0.285), there is always an error in strain indication if the transverse sensitivity of the strain gage is other than zero. In some instances, this error is small enough to be neglected. In others, it is not. The error due to transverse sensitivity for a strain gage oriented at any angle, in any strain field, on any material, can be expressed as:

$$n_\varepsilon = \frac{K_t\left(\frac{\varepsilon_t}{\varepsilon_a} + \nu_0\right)}{1 - \nu_0 K_t} \times 100 \qquad (5)$$

where: n_ε = the error as a percentage of the actual strain along the gage axis.

ν_0 = the Poisson's ratio of the material on which the manufacturer's gage factor, F, was measured (usually 0.285).

ε_a, ε_t = respectively, the actual strains parallel and perpendicular to the primary sensing axis of the gage.*

TEMPERATURE COMPENSATION

Total strain occurring at a point in a structure is made up of two components.

i) **Mechanical strain**: produced by external sources.

ii) **Apparent strain**: Strain induced by thermal effects including expansion of specimen, expansion of the gauge metal and changes in electrical resistance o the gauge.

Net temperature induced change in resistance in a strain gauge installation becomes,

$$\left(\frac{\Delta R}{R_0}\right)_{T/O} = \left[\beta_G + F_G\left(\frac{1+K_t}{1-\nu_0 K_t}\right)(\alpha_S - \alpha_G)\right]\Delta T \qquad (1)$$

where, in consistent units:

$\left(\frac{\Delta R}{R_0}\right)_{T/O}$ = unit change in resistance from the initial reference resistance, R_0, caused by change in temperature resulting in thermal output.

β_G = temperature coefficient of resistance of the grid conductor.

F_G = gage factor of the strain gage.†

K_t = transverse sensitivity of the strain gage.

ν_0 = Poisson's ratio (0.285) of the standard test material used in calibrating the gage for its gage factor.

$(\alpha_S - \alpha_G)$ = difference in thermal expansion coefficients of substrate and grid, respectively.

ΔT = temperature change from an arbitrary initial reference temperature.

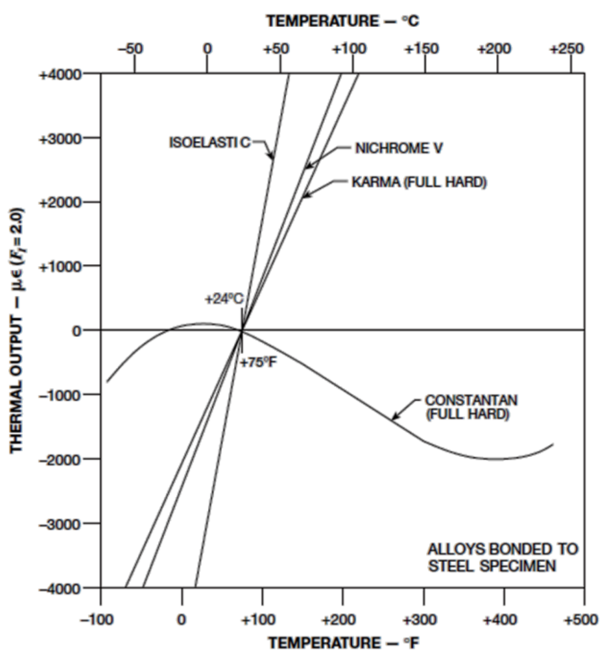


Fig: Thermal output variation with temperature for several strain gage alloys bonded to steel

Temperature Compensation Methods

i) Compensating (Dummy) Gage

The error due to thermal output can be completely eliminated by employing, in conjunction with the "active" strain gage, but connected in an adjacent arm of the Wheatstone bridge circuit, an identical compensating or "dummy" gage — mounted on an unstrained specimen made from the identical material as the test part, and subjected always to the same temperature as the active gage. Under these hypothetical conditions, the thermal outputs of the two gages should be identical. And, since identical resistance changes in adjacent arms of the Wheatstone bridge do not unbalance the circuit, the thermal outputs of the active and dummy strain gages should cancel exactly — leaving only the stress-induced strain in the active strain gage to be registered by the strain indicator. In this method, there is a difficulty in ensuring that the temperature of the compensating gage on the unstrained specimen is always identical to the temperature of the active gage.

For example, when strain measurements are to be made on a beam (Figure 1), which is thin enough so that under test conditions the temperatures on the two opposite surfaces normal to the plane of bending are the same, the two strain gages can be installed directly opposite each other on these surfaces. Similarly, for a bar in pure torsion (Figure 2), the two gages can be installed adjacent to each other and aligned along the principal axes of the bar (at 45° to the longitudinal axis).

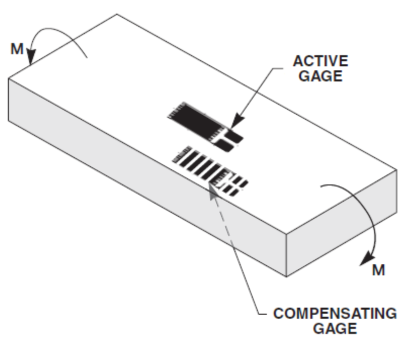


Figure 1

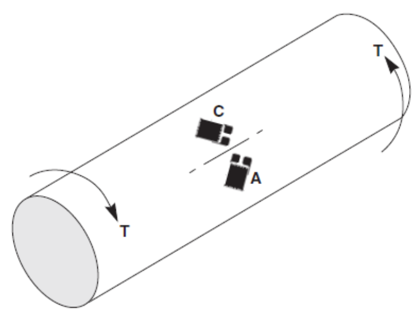


Figure 2

ii) Self-Temperature-Compensated Strain Gages

The metallurgical properties of certain strain gauge alloys - in particular, constantan and modified karma - are such that these alloys can be processed to minimize the thermal output over a wide temperature range when bonded to test materials with thermal expansion coefficients for which they are intended. Strain gages employing these specially processed alloys are referred to as *self-temperature-compensated.*

Since the advent of the self-temperature-compensated strain gage, the requirement for a matching unstrained dummy gage in the adjacent arm of the Wheatstone bridge has been relaxed considerably.

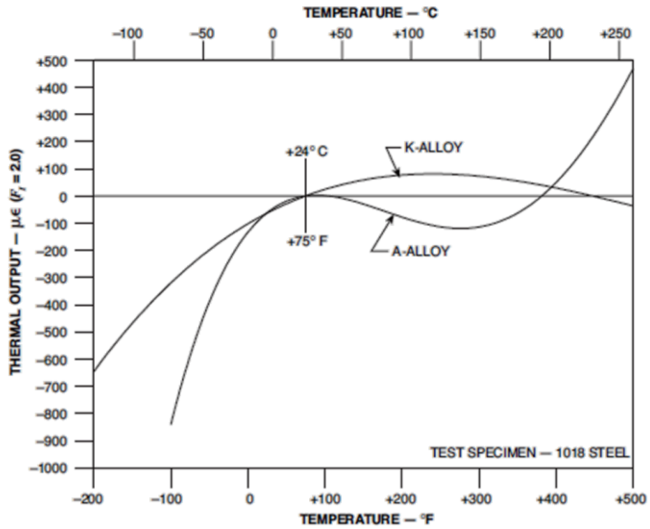


Fig: Typical thermal output variation with temperature for self-temperature-compensated constantan (A-alloy) and modified Karma (K-alloy) strain gages.

MODULE 3

STRAIN INDICATORS/RECORDERS

STRAIN INDICATORS/RECORDERS

- The output from a strain-sensing circuit, such as a Wheatstone bridge, potentiometer circuit, etc is quite small and varies from several micro volts to few mill volts.

- This small output has to be amplified before it can be indicated or recorded with conventional measuring instruments.

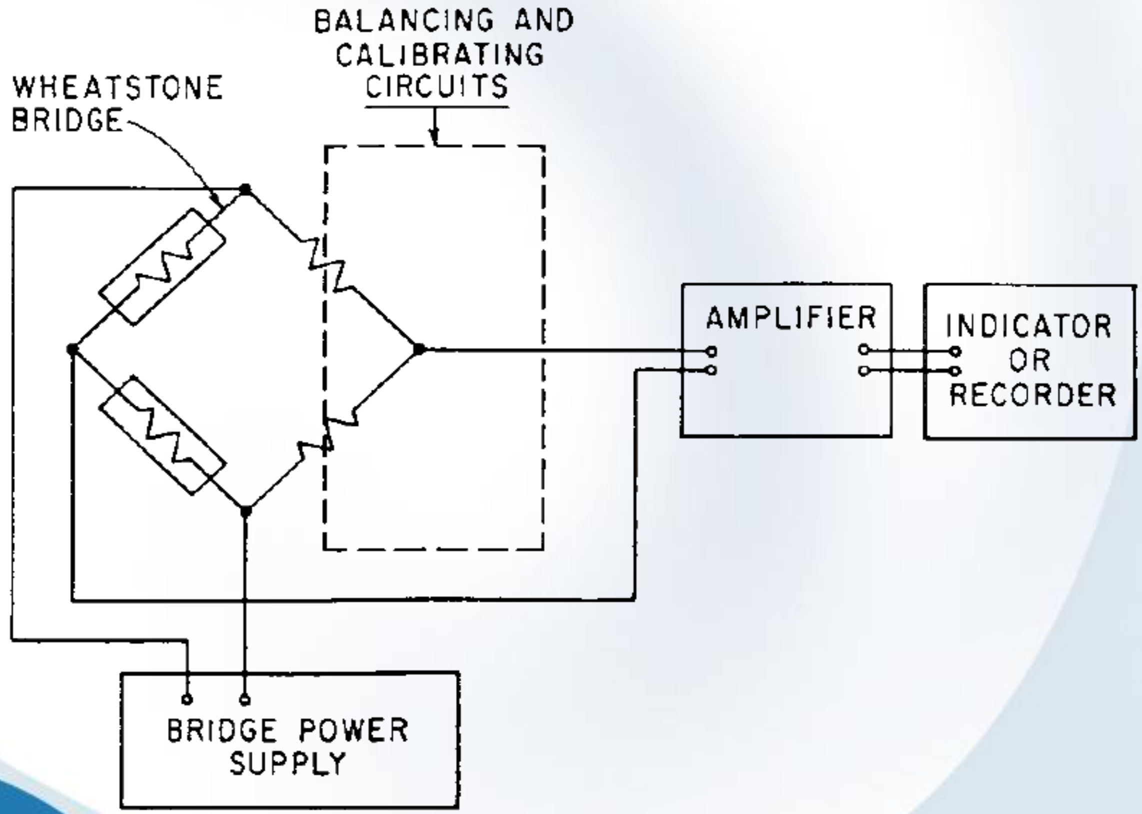
- This equipments may be broadly classified as:

  (i) Static strain-measuring instruments and

  (ii) Dynamic strain-measuring instruments

(a) DC Measuring System

The Wheatstone bridge is energized by a dc source and the output of the bridge is amplified by a highly stable dc amplifier before it is fed to an indicator or recorder.

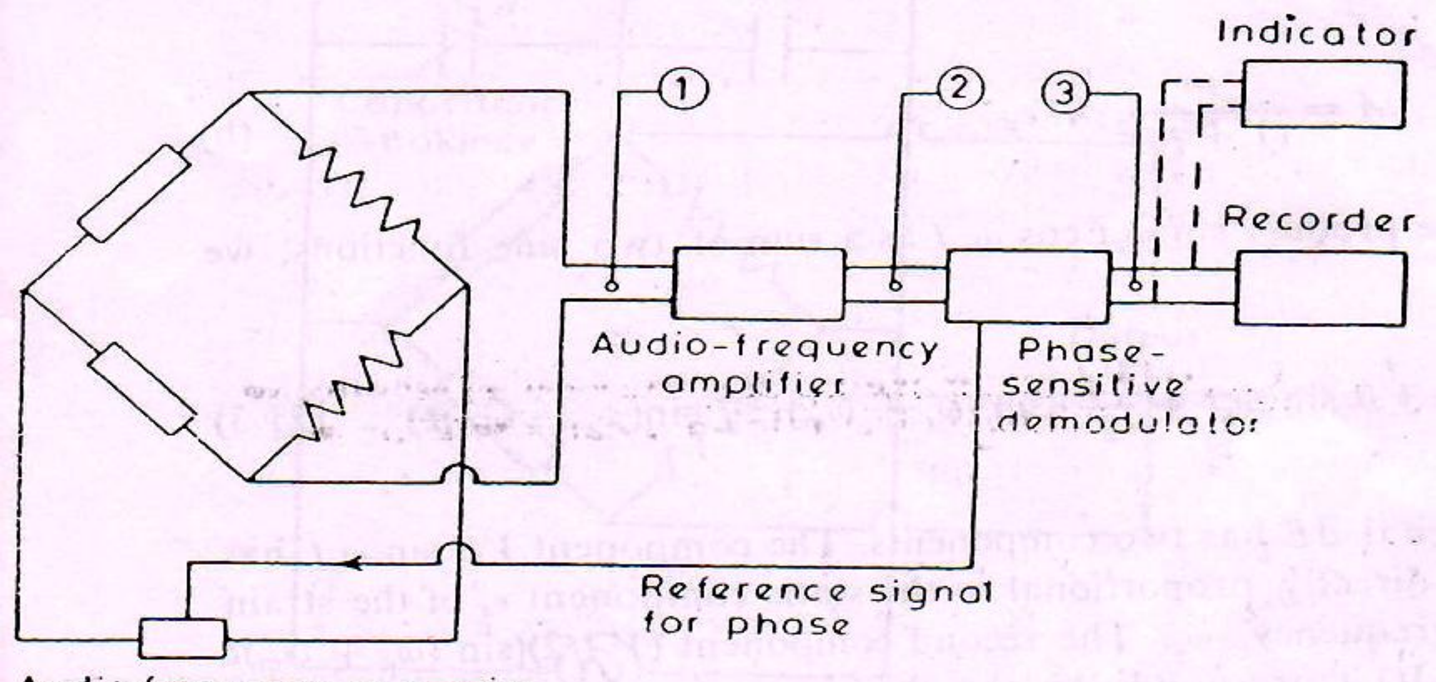


DC Measuring System

- A parallel-balancing circuit is generally provided for initial balancing of the bridge.
- A precision calibration resistor is connected through a switch across one of the arms of the bridge for shunt calibration of the system.
- output of the bridge after amplification may be fed to either an analog instrument, e.g. a micro voltmeter or to a digital indicator, e.g. digital voltmeter.

(b) Amplitude-Modulated Carrier-Wave System

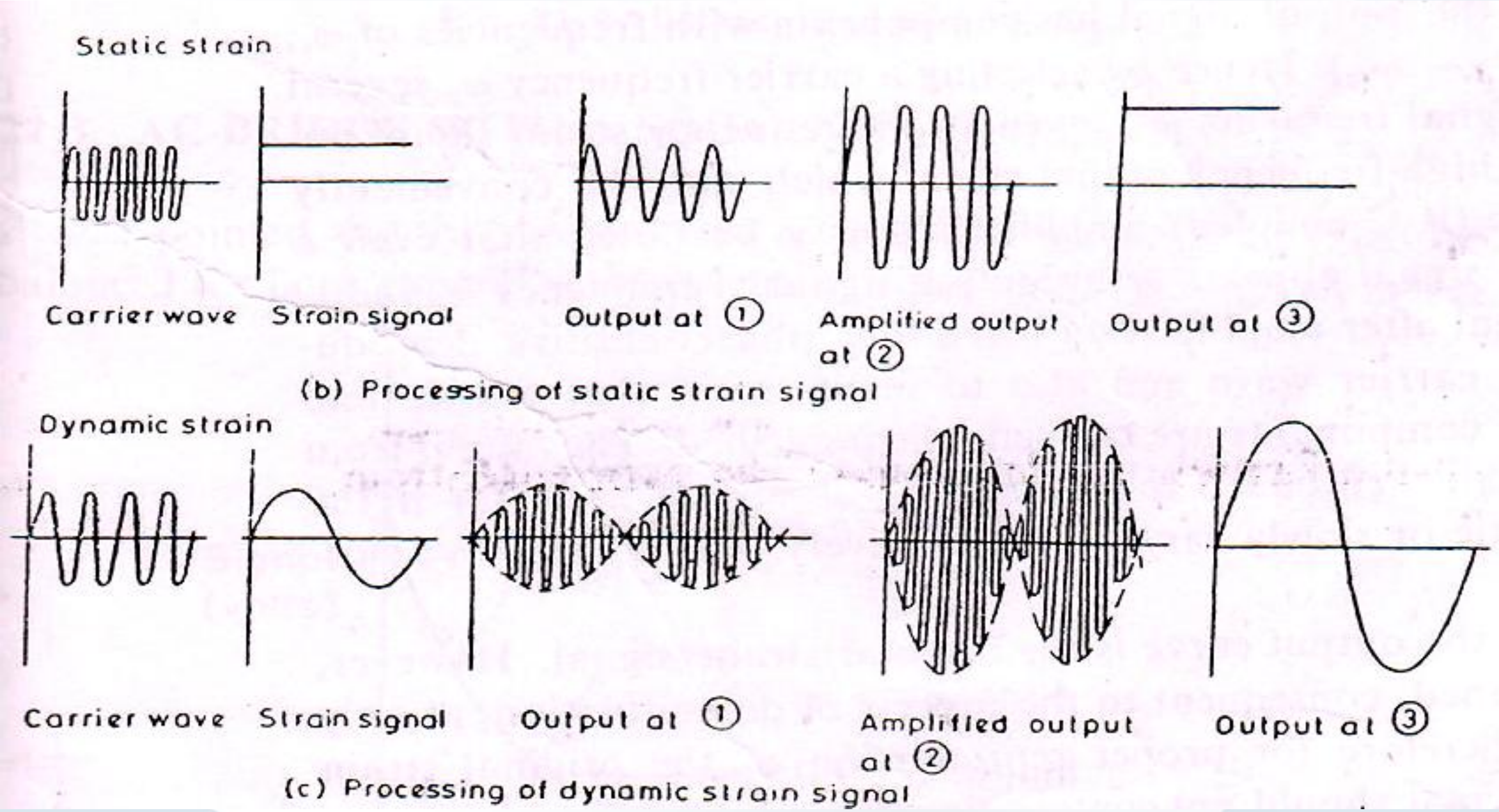
In this instead of a dc, ac voltage source is used to energize the Wheatstone bridge.



(a) Block diagram of an amplitude-modulated carrier-wave system

Amplitude-Modulated Carrier-Wave System

- The voltage source called the carrier is usually a sinusoidal wave with a frequency between 60Hz and 10 KHz.



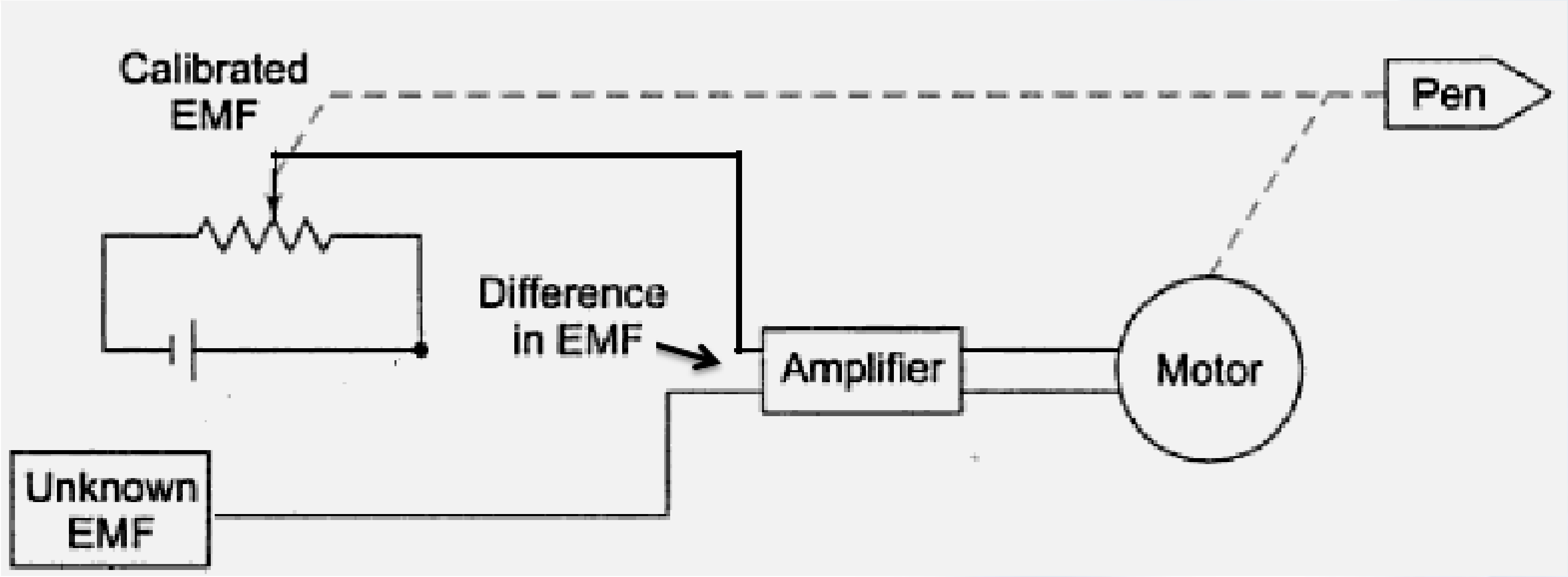
DYNAMIC STRAIN-MEASUREMENTS

- The type of indicator or recorder used is essentially dependent on the maximum frequency in the dynamic-strain signal.

(a) <u>Potentiometer and x y-recorders</u>

- For recording strain signals with very low frequency components (0 to 2 Hz), potentiometer recorders and X Y recorders are well-suited.
- The output voltage from Wheatstone bridge can be directly fed to these recorders.
- In these recorders a servo-driven system is used to give motion to the pen.

Potentiometer recorders



- The difference between the input signal and the potentiometer voltage is the error signal. This error signal is amplified and is used to energize the field coil of a dc motor.

Potentiometer recorders

- The motor is so connected that it turns in a direction that rotates the voltage divider arm (geared to it) in the direction that reduces the error.
- As the error becomes smaller, the motor slows down and finally stops at the point where the error is zero, thus producing the null balance.
- This is achieved by mechanically connecting the variable arm/wiper to the armature of the dc motor. The pen is also mechanically connected to the arm.

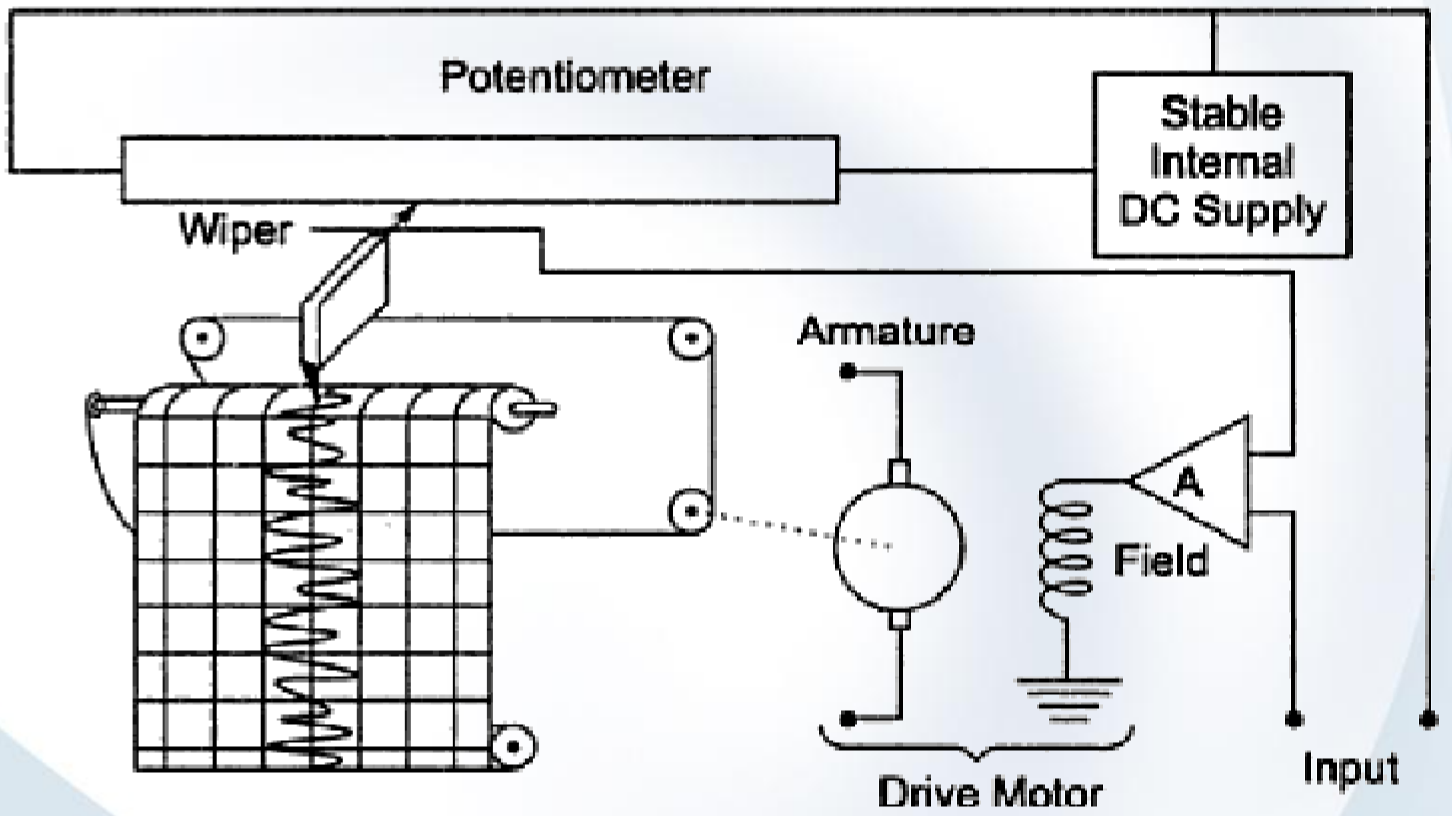
Potentiometer recorders

- Hence as the wiper moves in a particular direction, the pen also moves in synchronism in the same direction, thereby recording the input waveform.

- The wiper comes to rest when the unknown signal voltage is balanced against the voltage of the potentiometer.

Potentiometer recorders

Block diagram of self-balancing potentiometer recorder



x y-recorders

- In most research fields, it is often convenient to plot the instantaneous relationship between two variables y=f(x), rather than to plot each variable separately as a function of time. In such cases, the X-Y recorder is used.

- In an analog X-Y recorder, the writing head is deflected in either the x-direction or the y-direction on a fixed graph chart paper.

x y-recorders

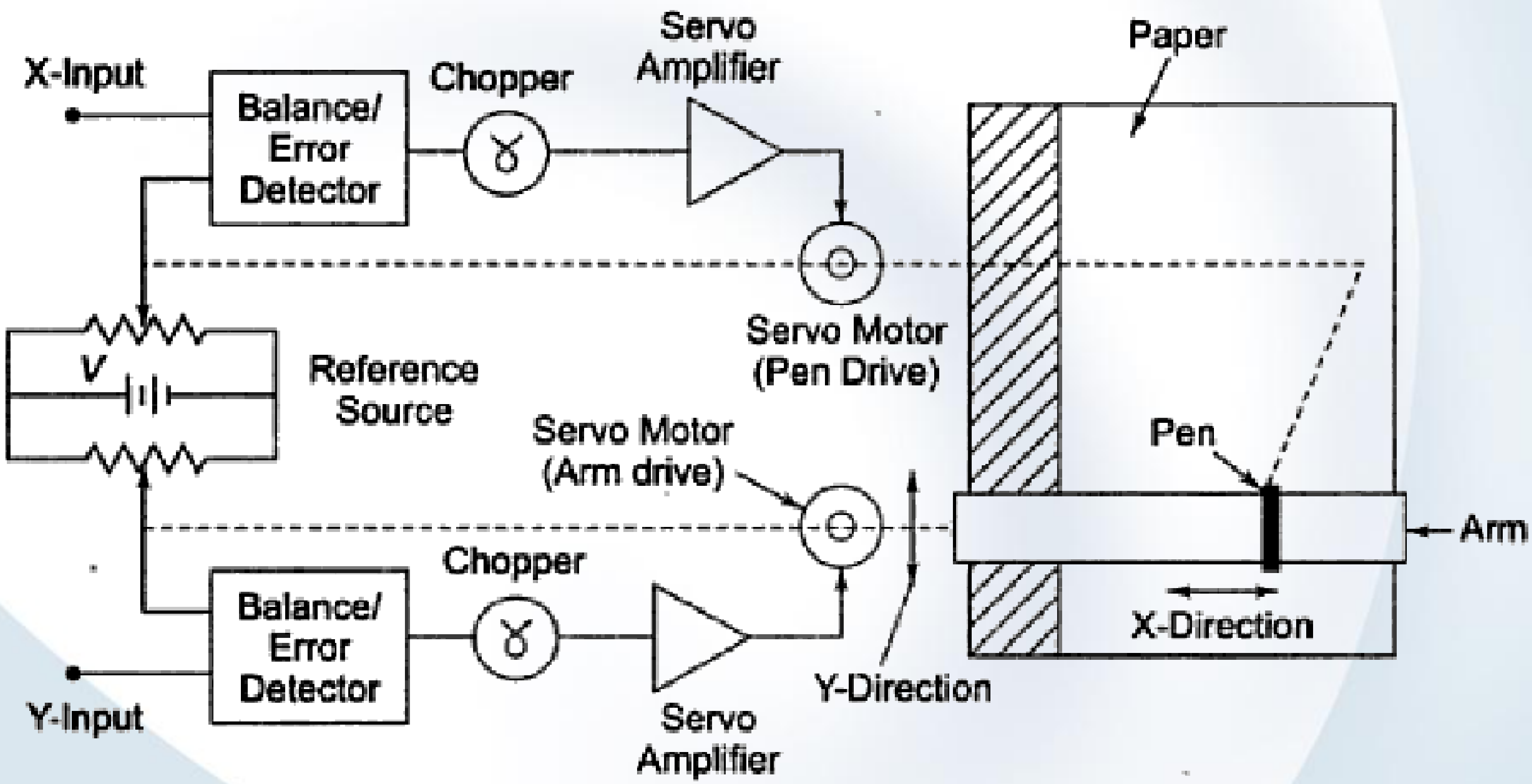
- The writing head is controlled by a servo feedback system or by a self balancing potentiometer and it consist of one or two pens, depending on the application.

- In practice, one emf is plotted as a function of another emf in an X-Y recorder.

- The motion of the recording pen in both the axis is driven by servo-system, with reference to a stationary chart paper.

- The movement in x and *y* directions is obtained through a sliding pen and moving arm arrangement.

A typical block diagram of an X-Y recorder



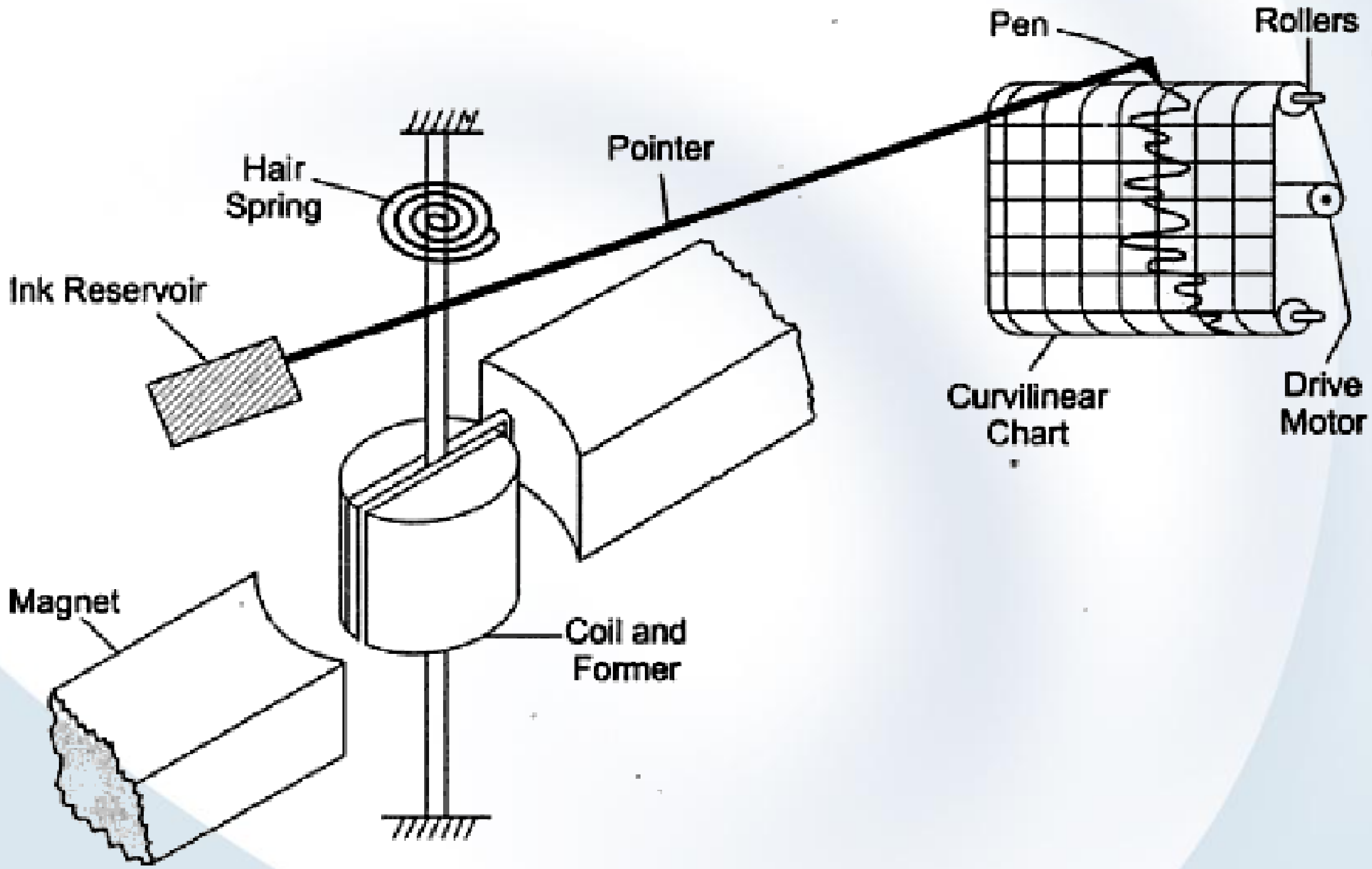
(b) Pen Recorders

- In these recorders, galvanometers are used to sense the input signal and deflect the arm carrying the writing pen by an amount directly proportional to the input signal.

- In the pen-writing oscillograph, a conventional pen traces in ink the record on paper. These recorders are suitable for recording signals with frequencies from zero to about 60 Hz.

Pen Recorders

Galvanometer type pen recorder



Pen Recorders

- The pointer deflects when current flows through the moving coil. The deflection of the pointer is directly proportional to the magnitude of the current flowing through the coil.

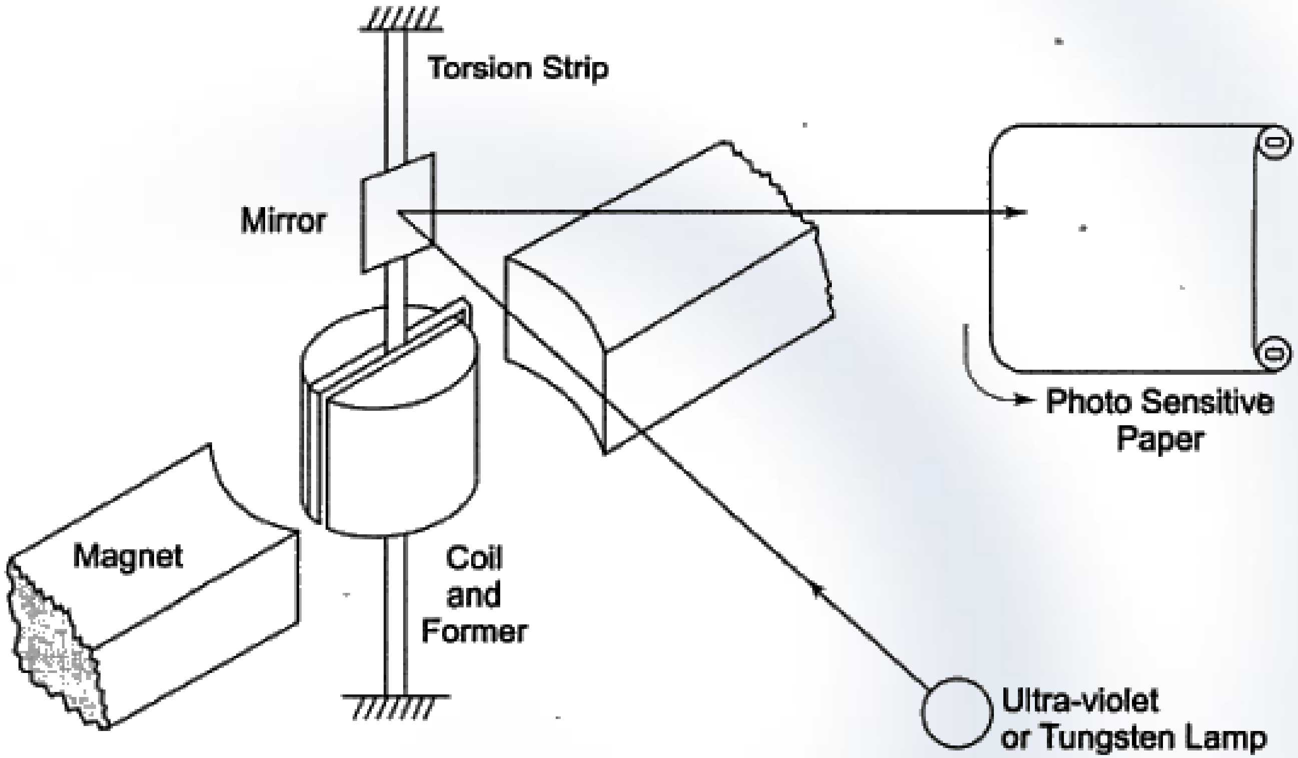
- As the position of the coil follows the variation of the signal current being recorded, the pen is accordingly deflected across the paper chart.

- The paper is pulled from a supply roll by a motor driven transport mechanism. Thus, as the paper moves past the pen and as the pen is deflected, the signal waveform is traced on the paper.

Pen Recorders

- The recording pen is connected to an ink reservoir through a narrow bore tube. Gravity and capillary action establish a flow of ink from the reservoir through the tubing and into the hollow of the pen.

(c) Light-Beam Recorders



Block diagram of light beam recorder

Light-Beam Recorders

- In pen recorders the inertia of the arm carrying the pen restricts their operation to frequencies less than about 100Hz.

- In light-beam recorders a weightless light beam deflected by the mirror of the galvanometer records the signal on a special light-sensitive paper.

- A current proportional to the input signal flows through the galvanometer and tilts the mirror mounted on the galvanometer. A light beam deflected by this mirror is focused or a light-sensitive paper on the rotating drum.

Light-Beam Recorders

- The deflection of the light beam is directly proportional to the strength of the input signal.
- With these recorders strain signals with frequencies in the range 0 to l0KHz can be recorded.
- Through the provision of multiple galvanometers, simultaneous and continuous recording of several signals can be achieved.

(d) Magnetic-Tape Recorders

- Magnetic-tape recorders are commercially available for recording and storing strain-gauge data of frequencies in the range 0 to about 100 kHz.
- Multiple signals can be recorded simultaneously. The main advantages of recording with magnetic-tape recorders are:

a) Both high sensitivity and high range are available.

b) Recorders on the magnetic tape can be erased if not required and the tape can be reused.

c) As the recording on the magnetic tape is electrical, it can be played back at any convenient time and displayed on an oscilloscope, magnified if necessary, with an extended or compressed time base.

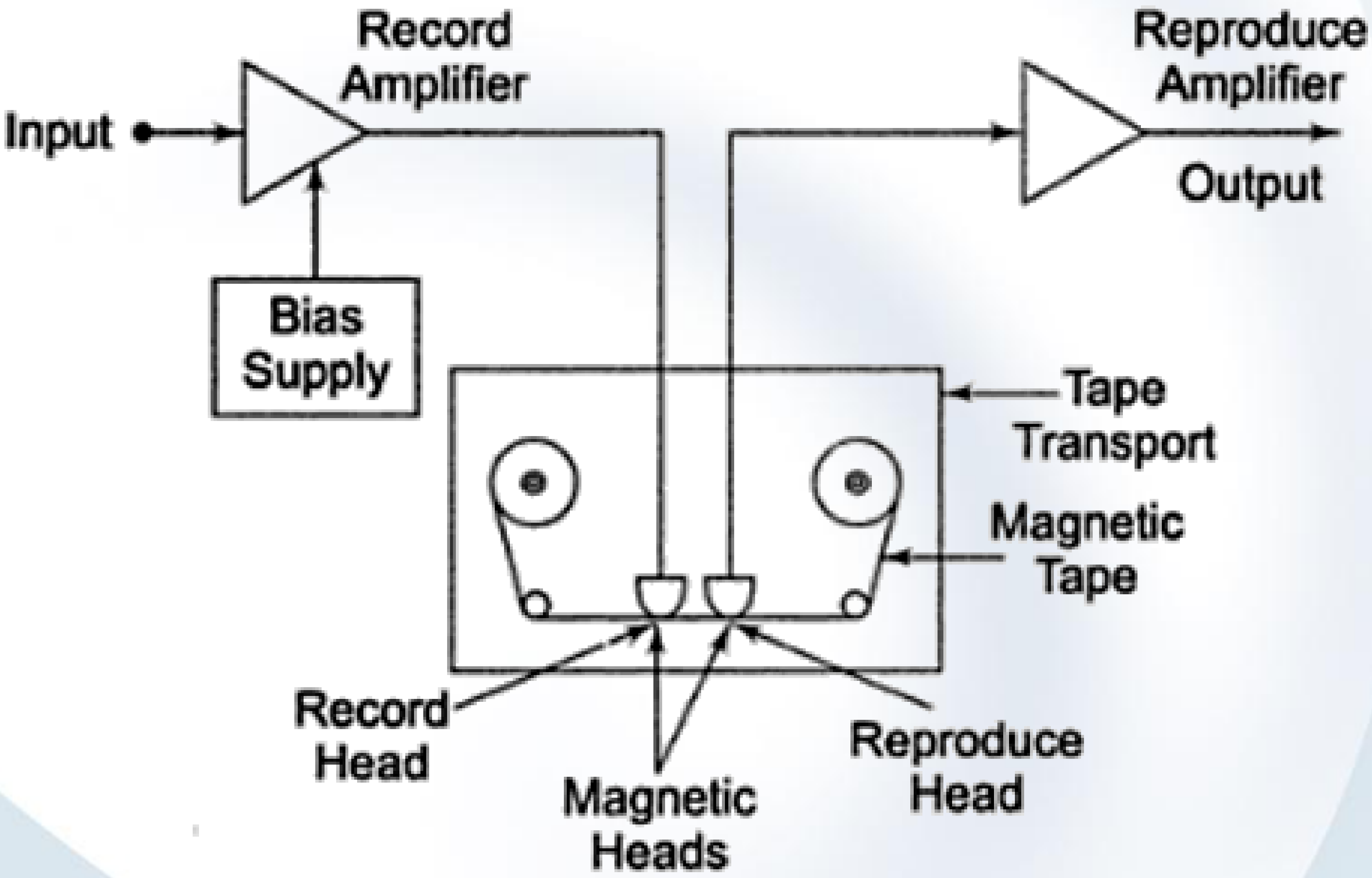
Magnetic-Tape Recorders

- Magnetic-tape analogue-data-recording systems are mostly of two types — the direct recording type and the type with frequency modulation (FM) system.

(i) Direct recording type:

- The magnetic tape is essentially a plastic film on the surface of which fine particles of magnetic material are dispersed uniformly.
- The current flowing through the winding in the record head is proportional to the input signal. The level of magnetism developed in the magnetic coating on the tape during recording is proportional input signal.

Magnetic-Tape Recorders

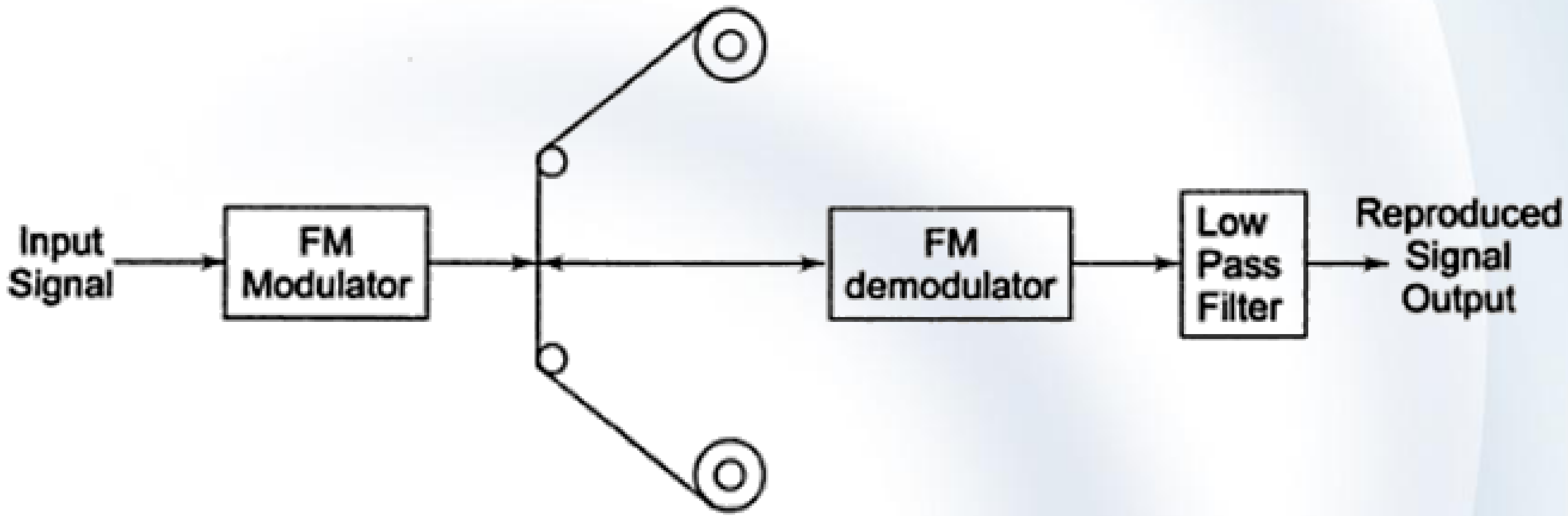


Magnetic-Tape Recorders

- The reproduce head is proportional to the changes in the level of magnetism over the recorded length of the tape.
- Through a suitable amplifier system, the signals from the reproduce head are processed and made suitable either to be recorded on an oscillograph recorder or to be further processed in a computer.
- The direct recording type of the recorder is cheaper but suitable only for recording dynamic signals without dc components.
- It is less accurate than the FM-type magnetic-tape recorder. Therefore it is mostly used for recording audio signals.

Magnetic-Tape Recorders

(ii) FM recorder



- In the FM-type recorder, the input signal modulates the frequency of the carrier wave. Deviations in the frequency of the carrier signal are pro-portional to the input signal.

Magnetic-Tape Recorders

- Thus both dc and ac components of the input signal can be recorded in this system to an accuracy much greater than that achievable in the direct recording system.

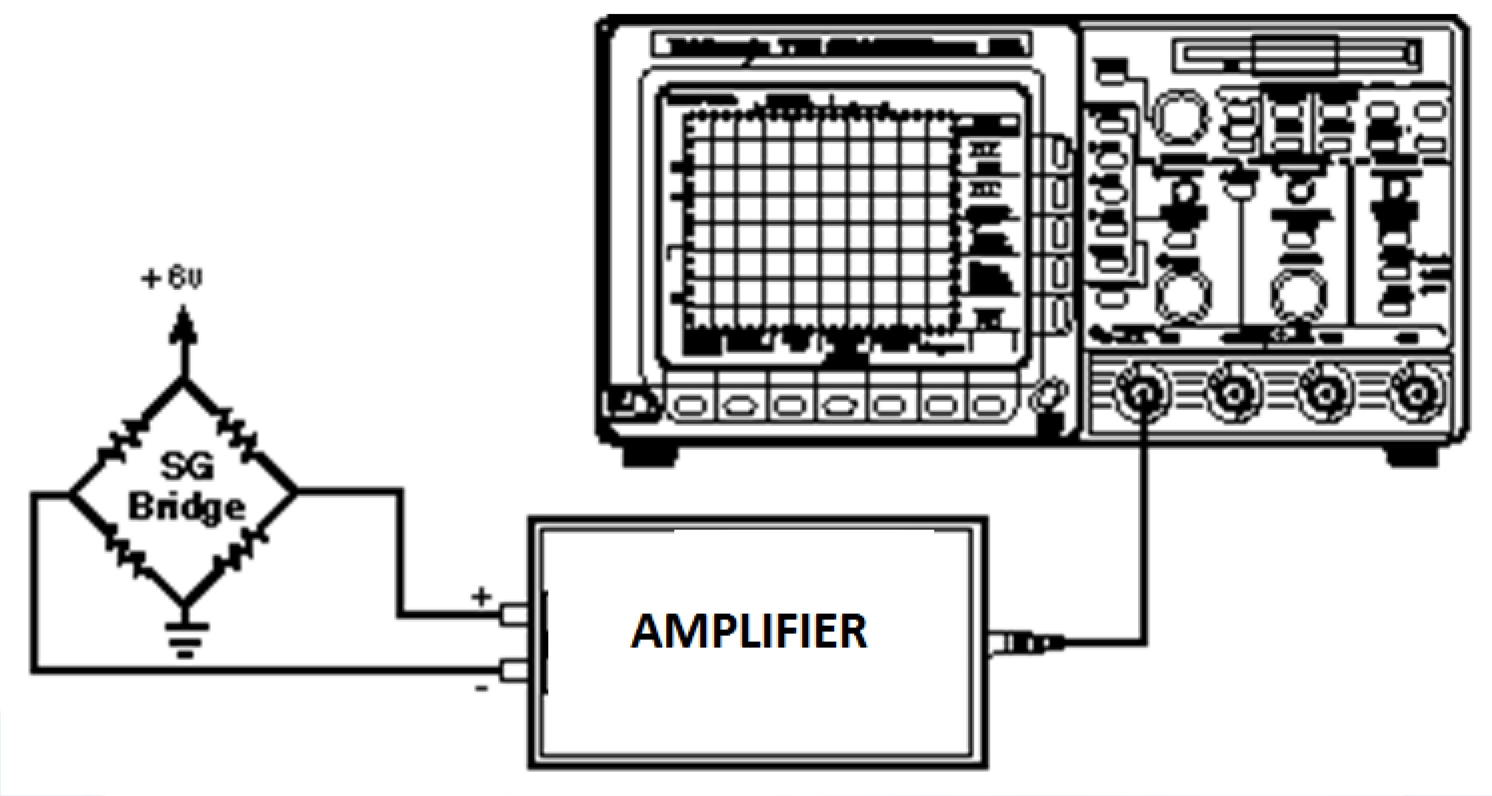
- In a digital tape recorder the analogue signal is first converted to the digital form before it is recorded on the magnetic tape. Data on a digital-tape recorder can be directly processed by a computer.

(e) Cathode-Ray Oscilloscope

- An attractive feature of a cathode-ray oscilloscope is its capability to record transient voltage signals with very high frequency components—signals with frequencies as high as 500 MHz can be recorded.

- The output signal from the Wheatstone bridge or the potentiometer circuit is first amplified using a high gain dc or ac amplifier and then fed usually to the y-plates of the oscilloscope so as to deflect the beam (spot) along the y-axis (vertical).

(e) Cathode-Ray Oscilloscope



Cathode-Ray Oscilloscope

- To provide a time base, a suitable signal usually generated within the oscilloscope is fed to the x-plates of the oscilloscope; this signal deflects the spot along the x-axis (horizontal).

- For studying phenomena of various durations, the time base can be varied over *a* wide range: 0.1s to say 5s per division on the oscilloscope screen.

- In a storage oscilloscope the signal is retained and, displayed on the screen of the oscilloscope well after the original signal has died down.

Strain Rosettes

MOHAMED SAMEER T K

Aero Dept. JCET

Strain Rosettes

When the state of strain at a point and the direction of principal strains is known, then the strain gauges can be oriented along these directions, and strain measurements may be made. However, when the state of strain is not known, then three or more gauges may be used at the point to determine the state of strain at the point. The resulting configuration is termed a strain rosette. Strain-rosette analysis is the art of arranging strain gauges as rosettes at a number of points on the object to be investigated, taking the measurements, and computing the state of stress at these points.

Strain Rosette Configurations

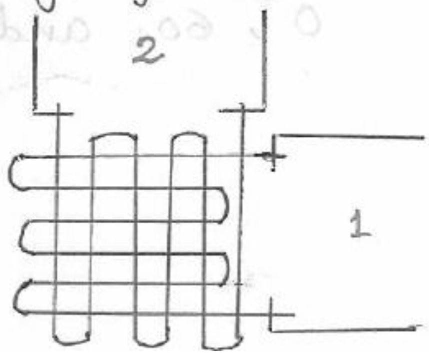
The strain gauges may be arranged in the following ways to obtain strain rosettes :

1. Two gauge rosette.
2. Rectangular rosette.
   (*a*) Three element ;
   (*b*) Four element.
3. Delta or Equiangular rosette.
4. T-Delta rosette.

In the two-gauge rosette the gauges 1 and 2 are arranged at 0° at 90° respectively. In the rectangular rosette the gauges 1, 2 and 3 are arranged at 0°, 45° and 90° respectively. Delta rosette has gauges 1, 2, 3 arranged at 0°, 60° and 120° respectively, whereas in the T-delta, the gauges 1, 2, 3, 4 are arranged at 0°, 60°, 120° and 90° respectively.

Strain Rosette Configurations

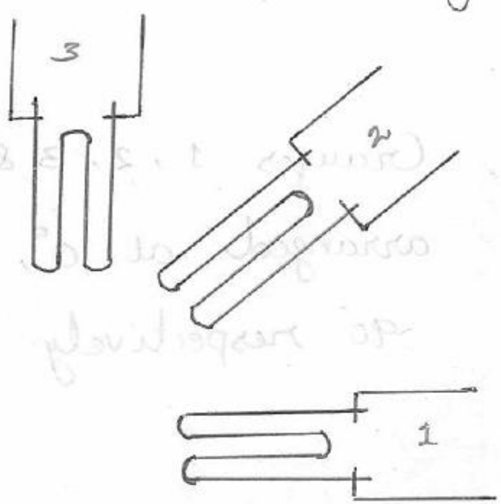
1. Two gauge rosette.



Strain gauges 1 and 2 are arranged at 0° and 90° respectively.

2. Rectangular rosette.

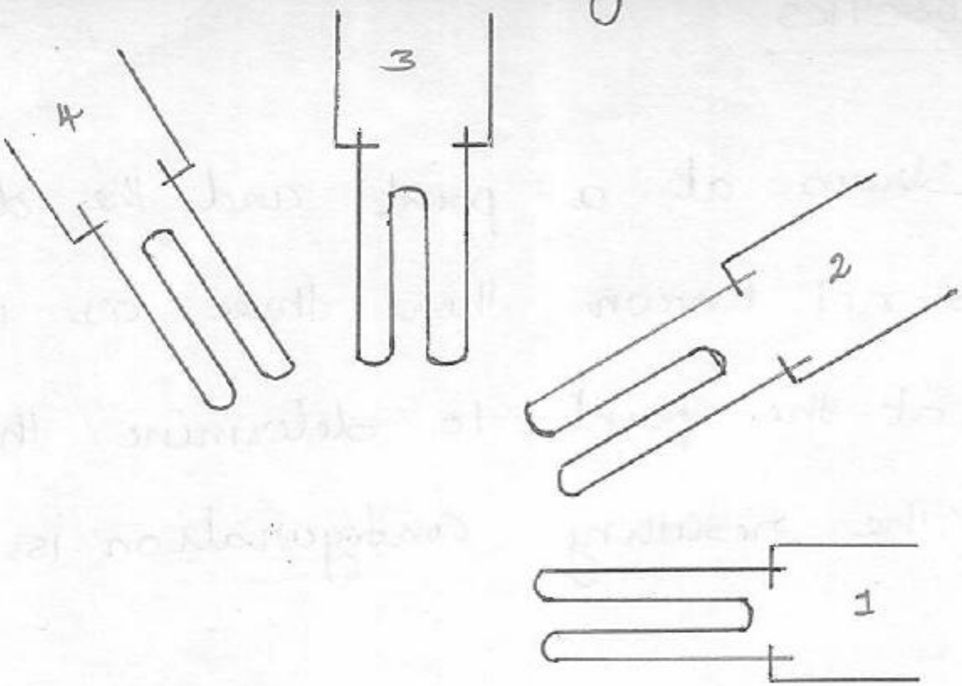
(a) Three element rectangular rosette.



Gauges 1, 2 and 3 are arranged at 0°, 45° and 90° respectively.

Strain Rosette Configurations

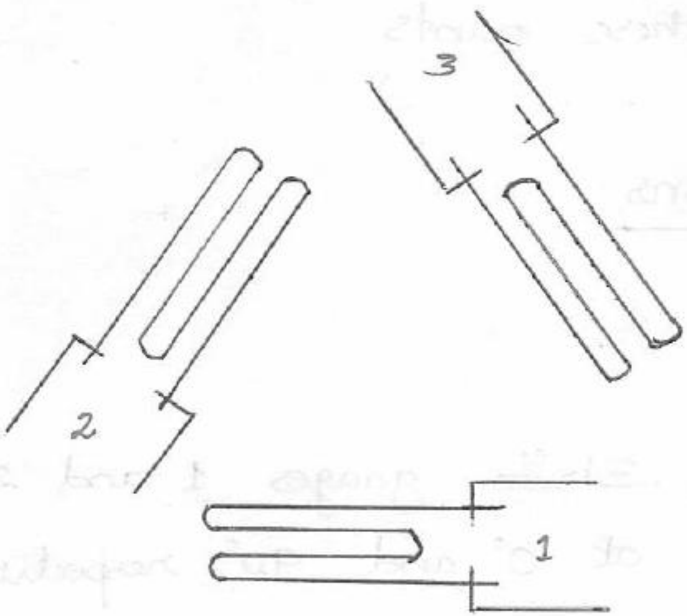
(b) Four element rectangular rosette.



Gauges are arranged at 0°, 45°, 90° and 135° respectively.

Strain Rosette Configurations

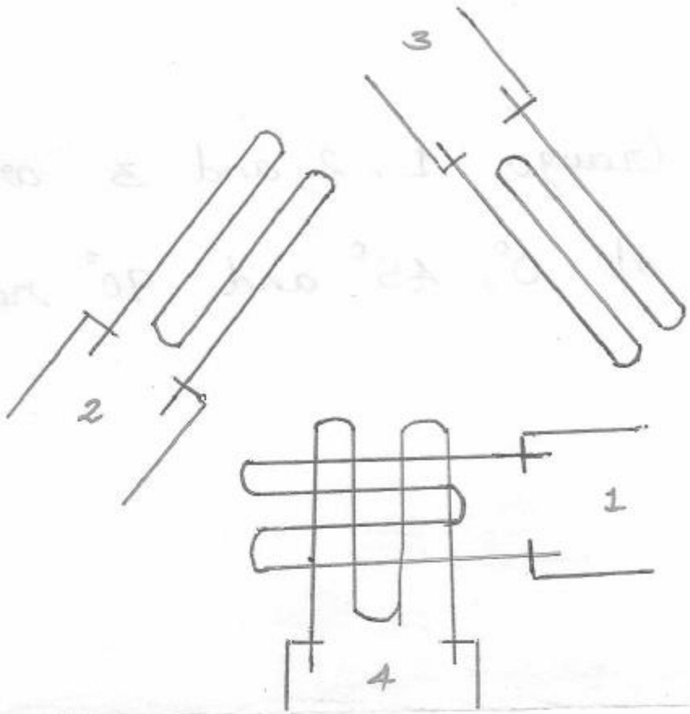
3. Delta Rosette



Gauges are arranged at 0°, 60° and 120° respectively.

Strain Rosette Configurations

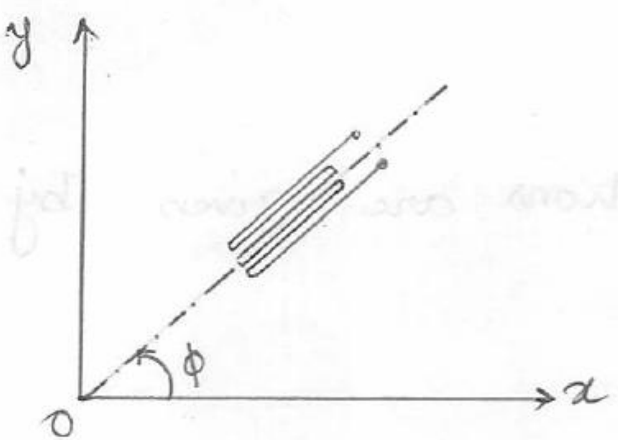
4. T - Delta Rosette

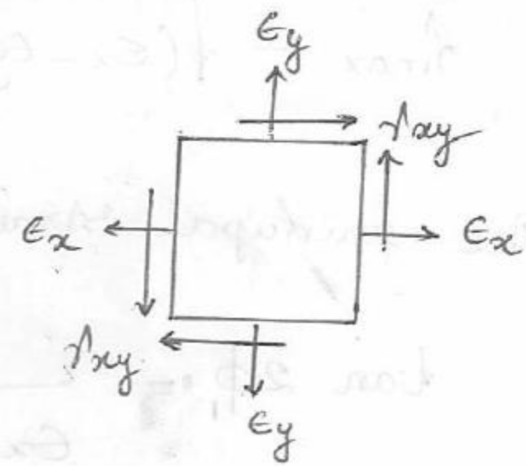


Gauges 1, 2, 3 & 4 are arranged at 0°, 60°, 120° and 90° respectively.

Stress-Strain Relationship

Consider a strain gauge situated at an angular position ϕ measured anti-clock wise from direction x in a plane-strain field.





Stress-Strain Relationship

The strain measured by the gauge is given by

$$\epsilon_\phi = \epsilon_x \cos^2\phi + \epsilon_y \sin^2\phi + \gamma_{xy} \sin\phi \cos\phi$$

$$= \frac{\epsilon_x}{2}(1+\cos 2\phi) + \frac{\epsilon_y}{2}(1-\cos 2\phi) + \frac{\gamma_{xy}}{2} \sin 2\phi$$

$$= \frac{1}{2}(\epsilon_x + \epsilon_y) + \frac{1}{2}(\epsilon_x - \epsilon_y)\cos 2\phi + \frac{\gamma_{xy}}{2} \sin 2\phi \qquad \text{--- (1)}$$

Stress-Strain Relationship

equation ① contains three unknowns, ϵ_x, ϵ_y and γ_{xy}.

∴ 3 strains ϵ_a, ϵ_b and ϵ_c are required along three directions a, b & c to determine the state of strain at a point.

∴

$$\epsilon_a = \frac{1}{2}(\epsilon_x + \epsilon_y) + \frac{1}{2}(\epsilon_x - \epsilon_y)\cos 2\phi_a + \frac{\gamma_{xy}}{2}\sin 2\phi_a$$

$$\epsilon_b = \frac{1}{2}(\epsilon_x + \epsilon_y) + \frac{1}{2}(\epsilon_x - \epsilon_y)\cos 2\phi_b + \frac{\gamma_{xy}}{2}\sin 2\phi_b$$

$$\epsilon_c = \frac{1}{2}(\epsilon_x + \epsilon_y) + \frac{1}{2}(\epsilon_x - \epsilon_y)\cos 2\phi_c + \frac{\gamma_{xy}}{2}\sin 2\phi_c$$

From the above equations, ϵ_x, ϵ_y and γ_{xy} can be evaluated

Stress-Strain Relationship

The principal strains are then.

$$\epsilon_{1,2} = \frac{1}{2}(\epsilon_x + \epsilon_y) \pm \frac{1}{2}\sqrt{(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2}$$

and maximum shear strain

$$\gamma_{max} = \sqrt{(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2}$$

Stress-Strain Relationship

The principal strain directions are given by

$$\tan 2\phi = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

$$\phi_1 = \frac{1}{2}\tan^{-1}\left[\frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}\right]$$

$$\phi_2 = 90^\circ + \phi_1$$

where ϕ_1 and ϕ_2 are the angles between principal strains ϵ_1 & ϵ_2 respectively.

Stress-Strain Relationship

The principal stresses are then become

$$\sigma_1 = \frac{E}{1-\nu^2}\left(\epsilon_1 + \nu\epsilon_2\right)$$

$$\sigma_2 = \frac{E}{1-\nu^2}\left(\epsilon_2 + \nu\epsilon_1\right)$$

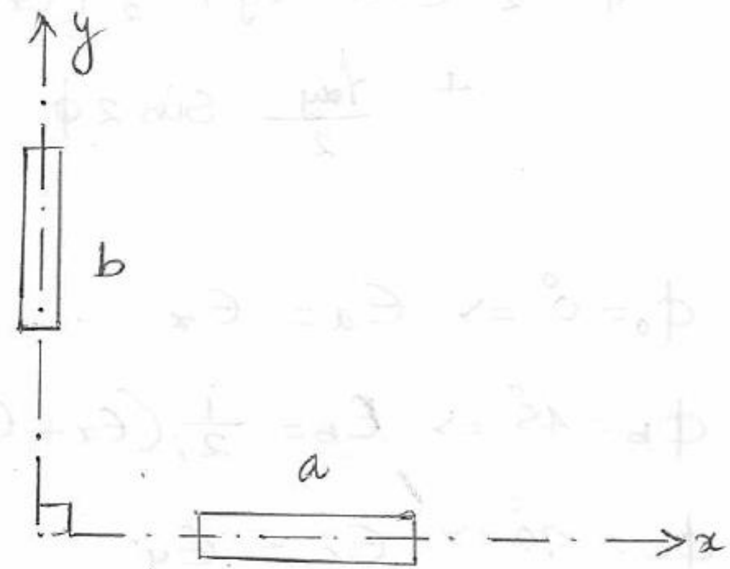
The maximum shear stress is,

$$\tau_{max} = \frac{E}{2(1+\nu)}\gamma_{max}.$$

Analytical Solutions

(i) Two gauge rosette:

This rosette is suitable only when the directions of principal strain are known.



The gauge 'a' is arranged along the maximum strain direction chosen along the x-axis so that $\phi_a = 0^\circ$ and the gauge 'b' is set along the minimum strain direction so that $\phi_b = 90^\circ$.

Analytical Solutions

$$\epsilon_\phi = \frac{1}{2}(\epsilon_x + \epsilon_y) + \frac{1}{2}(\epsilon_x - \epsilon_y)\cos 2\phi + \frac{\gamma_{xy}}{2}\sin 2\phi$$

$$\phi_a = 0 \Rightarrow \epsilon_a = \epsilon_x$$

$$\phi_b = 90^\circ \Rightarrow \epsilon_b = \epsilon_y$$

$$\epsilon_{1,2} = \frac{1}{2}(\epsilon_x + \epsilon_y) \pm \frac{1}{2}\sqrt{(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2}$$

$$\therefore \epsilon_1 = \epsilon_a \text{ and } \epsilon_2 = \epsilon_b$$

$$\gamma_{max} = \sqrt{(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2}$$

Analytical Solutions

$$\gamma_{max} = \epsilon_a - \epsilon_b$$

$$\sigma_1 = \frac{E}{1-\nu^2}(\epsilon_a + \nu\epsilon_b)$$

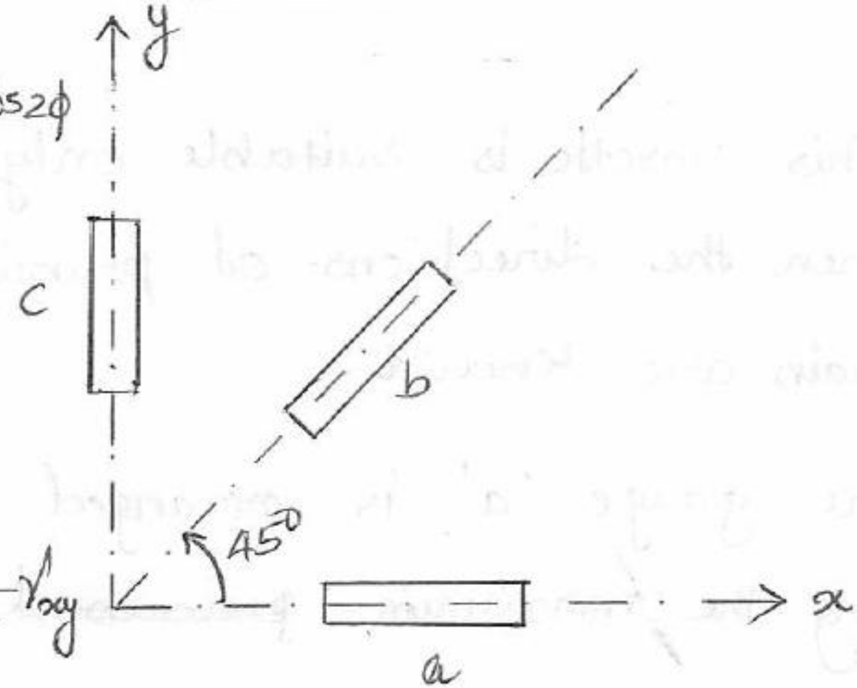
$$\sigma_2 = \frac{E}{1-\nu^2}(\epsilon_b + \nu\epsilon_a)$$

$$\tau_{max} = \frac{E}{2(1+\nu)}(\epsilon_a - \epsilon_b)$$

Analytical Solutions

(b)(i) Three element rectangular rosette

$$\epsilon_\phi = \frac{1}{2}(\epsilon_x + \epsilon_y) + \frac{1}{2}(\epsilon_x - \epsilon_y)\cos 2\phi + \frac{\gamma_{xy}}{2}\sin 2\phi$$



$\phi_a = 0^\circ \Rightarrow \epsilon_a = \epsilon_x$

$\phi_b = 45^\circ \Rightarrow \epsilon_b = \frac{1}{2}(\epsilon_x + \epsilon_y) + \frac{1}{2}\gamma_{xy}$

$\phi_c = 90^\circ \Rightarrow \epsilon_c = \epsilon_y$

Analytical Solutions

solving ϵ_x, ϵ_y & γ_{xy}, we get

$$\epsilon_x = \epsilon_a$$

$$\epsilon_y = \epsilon_c$$

$$\gamma_{xy} = 2\epsilon_b - (\epsilon_a + \epsilon_c)$$

The principal strains are given by

$$\epsilon_{1,2} = \frac{1}{2}(\epsilon_x + \epsilon_y) \pm \frac{1}{2}\sqrt{(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2}$$

$$= \frac{1}{2}(\epsilon_a + \epsilon_c) \pm \frac{1}{2}\sqrt{(\epsilon_a - \epsilon_c)^2 + (2\epsilon_b - \epsilon_a - \epsilon_c)^2}$$

Analytical Solutions

Maximum shear strain becomes

$$\gamma_{max} = \sqrt{(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2}$$

$$= \sqrt{(\epsilon_a - \epsilon_c)^2 + (2\epsilon_b - \epsilon_a - \epsilon_c)^2}$$

principal strain directions are

$$\phi_1 = \frac{1}{2}\tan^{-1}\left[\frac{2\epsilon_b - \epsilon_a - \epsilon_c}{\epsilon_a - \epsilon_c}\right]$$

Analytical Solutions

The principal stresses are,

$$\sigma_1 = \frac{E}{1-\nu^2}\left(\epsilon_1 + \nu\,\epsilon_2\right)$$

$$= \frac{E}{1-\nu^2}\cdot\frac{1}{2}\left[(\epsilon_a + \epsilon_c) + \sqrt{(\epsilon_a - \epsilon_c)^2 + (2\epsilon_b - \epsilon_a - \epsilon_c)^2}\right.$$
$$\left. + \nu(\epsilon_a + \epsilon_c) - \nu\sqrt{(\epsilon_a - \epsilon_c)^2 + (2\epsilon_b - \epsilon_a - \epsilon_c)^2}\right]$$

$$= \frac{E}{1-\nu^2}\cdot\frac{1}{2}\left[(\epsilon_a + \epsilon_c)(1+\nu) + \sqrt{(\epsilon_a - \epsilon_c)^2 + (2\epsilon_b - \epsilon_a - \epsilon_c)^2}\,(1-\nu)\right]$$

Analytical Solutions

$$= \frac{E}{2}\left[\frac{\epsilon_a+\epsilon_c}{1-\nu}+\frac{1}{1+\nu}\sqrt{(\epsilon_a-\epsilon_c)^2+(2\epsilon_b-\epsilon_a-\epsilon_c)^2}\right]$$

$$\sigma_2 = \frac{E}{1-\nu^2}(\epsilon_2+\nu\epsilon_1)$$

$$= \frac{E}{2}\left[\frac{\epsilon_a+\epsilon_c}{1-\nu}-\frac{1}{1+\nu}\sqrt{(\epsilon_a-\epsilon_c)^2+(2\epsilon_b-\epsilon_a-\epsilon_c)^2}\right]$$

Analytical Solutions

maximum shear stress,

$$\tau_{max} = \frac{E}{2(1+\nu)} \gamma_{max}.$$

$$= \frac{E}{2(1+\nu)} \sqrt{(\epsilon_a - \epsilon_c)^2 + (2\epsilon_b - \epsilon_a - \epsilon_c)^2}$$

Analytical Solutions

Example 4.2 *The following readings of strain were obtained on a rectangular strain rosette mounted on aluminium for which E = 70 GPa,* $\nu = 0.32$, $\varepsilon_a = 285 \times 10^{-6}$, $\varepsilon_0 = 65 \times 10^{-6}$, $\varepsilon_c = 102 \times 10^{-6}$. *Determine the principal strains, principal strain directions, principal stresses and maximum shear stress.*

Solution. For a three-element rectangular rosette, we have

$$\varepsilon_{1,2} = \frac{1}{2}(\varepsilon_a + \varepsilon_c) \pm \frac{1}{2}\sqrt{(\varepsilon_a - \varepsilon_c)^2 + (2\,\varepsilon_b - \varepsilon_a - \varepsilon_c)^2}$$

$$= \frac{1}{2}\left[(285 + 102) \pm \sqrt{(285 - 102)^2 + (130 - 285 - 102)^2}\right] \times 10^{-6}$$

$$= 351.25 \times 10^{-6},\ 35.75 \times 10^{-6}$$

$$\gamma_{max} = \sqrt{(\varepsilon_a - \varepsilon_c)^2 + (2\,\varepsilon_b - \varepsilon_a - \varepsilon_c)^2}$$

$$= 315.5 \times 10^{-6} \text{ rad.}$$

$$\phi_1 = \frac{1}{2} \tan^{-1}\left(\frac{2\,\varepsilon_b - \varepsilon_a - \varepsilon_c}{\varepsilon_c - \varepsilon_c}\right)$$

$$= \frac{1}{2} \tan^{-1}\left(\frac{130 - 285 - 102}{285 - 102}\right)$$

$$= \frac{1}{2} \tan^{-1}(-1.4037)$$

$$\phi_1 = -(27^\circ\ 16')$$

$$\phi_2 = -(117^\circ\ 16')$$

$$\sigma_1 = \frac{E}{1 - \nu^2}(\varepsilon_1 + \nu\,\varepsilon_2)$$

$$= \frac{70 \times 10^9}{1 - (0.32)^2}(351.25 + 0.32 \times 35.75) \times 10^{-6}$$

$$= 28.285 \text{ MPa}$$

$$\sigma_2 = \frac{E}{1 - \nu^2}(\varepsilon_2 + \nu\,\varepsilon_1)$$

$$= \frac{70 \times 10^9}{1 - (0.32)^2}(35.75 + 0.32 \times 351.25) \times 10^{-6}$$

$$= 11.553 \text{ MPa}$$

$$\tau_{max} = \frac{E}{2(1 - \nu)}\gamma_{max} = \frac{70 \times 10^9}{2(1 + 0.32)} \times 315.5 \times 10^{-6}$$

$$= 8.365 \text{ MPa}$$

Analytical Solutions

(ii) Four element rectangular rosette.

$$\epsilon_\phi = \frac{1}{2}(\epsilon_x + \epsilon_y) + \frac{1}{2}(\epsilon_x - \epsilon_y)\cos 2\phi + \frac{\gamma_{xy}}{2}\sin 2\phi$$

$$\phi_a = 0^\circ \Rightarrow \epsilon_a = \epsilon_x$$

$$\phi_b = 45^\circ \Rightarrow \epsilon_b = \frac{1}{2}(\epsilon_x + \epsilon_y + \gamma_{xy})$$

$$\phi_c = 90^\circ \Rightarrow \epsilon_c = \epsilon_y.$$

$$\phi_d = 135^\circ \Rightarrow \epsilon_d = \frac{1}{2}(\epsilon_x + \epsilon_y - \gamma_{xy})$$

Analytical Solutions

Solving for ϵ_x, ϵ_y and γ_{xy}, we get

$$\epsilon_x = \epsilon_a$$

$$\epsilon_y = \epsilon_c$$

$$\gamma_{xy} = \epsilon_b - \epsilon_d.$$

$$\epsilon_{1,2} = \frac{1}{2}(\epsilon_a + \epsilon_c) \pm \frac{1}{2}\sqrt{(\epsilon_a - \epsilon_c)^2 + (\epsilon_b - \epsilon_d)^2}$$

$$\gamma_{max} = \sqrt{(\epsilon_a - \epsilon_c)^2 + (\epsilon_b - \epsilon_d)^2}$$

$$\phi_1 = \frac{1}{2}\tan^{-1}\left[\frac{\epsilon_b - \epsilon_d}{\epsilon_a - \epsilon_c}\right]$$

Analytical Solutions

$$\sigma_{1,2} = \frac{E}{2}\left[\frac{\epsilon_a+\epsilon_c}{1-\nu} \pm \frac{1}{1+\nu}\sqrt{(\epsilon_a-\epsilon_c)^2+(\epsilon_b-\epsilon_d)^2}\right]$$

$$\tau_{max} = \frac{E}{2(1+\nu)}\sqrt{(\epsilon_a-\epsilon_c)^2+(\epsilon_b-\epsilon_d)^2}$$

Analytical Solutions

(c) Deta Rosette

$$\phi_a = 0^\circ \Rightarrow \epsilon_a = \epsilon_x$$

$$\phi_b = 60^\circ \Rightarrow \epsilon_b = \frac{1}{2}(\epsilon_x + \epsilon_y) + \frac{1}{2}(\epsilon_x - \epsilon_y) \times -\frac{1}{2} + \frac{1}{2}\gamma_{xy} \cdot \frac{\sqrt{3}}{2}$$

$$\epsilon_b = \frac{1}{4}\left[\epsilon_x + 3\epsilon_y + \sqrt{3}\,\gamma_{xy}\right]$$

$$\phi_c = 120^\circ \Rightarrow \epsilon_c = \frac{1}{2}(\epsilon_x + \epsilon_y) + \frac{1}{2}(\epsilon_x - \epsilon_y) \times -\frac{1}{2} + \frac{1}{2}\gamma_{xy} \times -\frac{\sqrt{3}}{2}.$$

$$\epsilon_c = \frac{1}{4}\left[\epsilon_x + 3\epsilon_y - \sqrt{3}\,\gamma_{xy}\right]$$

Analytical Solutions

$$\epsilon_x = \epsilon_a$$

$$\epsilon_y = \frac{1}{3}\left(-\epsilon_a + 2\epsilon_b + 2\epsilon_c\right)$$

$$\gamma_{xy} = \frac{2}{\sqrt{3}}\left(\epsilon_b - \epsilon_c\right)$$

$$\epsilon_{1,2} = \frac{1}{2}\left[\epsilon_a + \frac{1}{3}\left(-\epsilon_a + 2\epsilon_b + 2\epsilon_c\right)\right] \pm \frac{1}{2}\sqrt{\left[\epsilon_a - \frac{1}{3}\left(-\epsilon_a + 2\epsilon_b + 2\epsilon_c\right)\right]^2 + \left(\frac{2(\epsilon_b - \epsilon_c)}{\sqrt{3}}\right)^2}$$

Analytical Solutions

$$= \frac{\epsilon_a + \epsilon_b + \epsilon_c}{3} \pm \frac{1}{2}\sqrt{\left(\frac{4}{3}\epsilon_a - \frac{2}{3}\epsilon_b - \frac{2}{3}\epsilon_c\right)^2 + \left(\frac{2(\epsilon_b - \epsilon_c)}{\sqrt{3}}\right)^2}$$

$$= \frac{\epsilon_a + \epsilon_b + \epsilon_c}{3} \pm \frac{1}{2}\sqrt{4\left(\frac{2\epsilon_a - \epsilon_b - \epsilon_c}{3}\right)^2 + 4\left(\frac{\epsilon_b - \epsilon_c}{\sqrt{3}}\right)^2}$$

Analytical Solutions

$$\epsilon_{1,2} = \frac{\epsilon_a+\epsilon_b+\epsilon_c}{3} \pm \sqrt{\left(\frac{2\epsilon_a-\epsilon_b-\epsilon_c}{3}\right)^2+\left(\frac{\epsilon_b-\epsilon_c}{\sqrt{3}}\right)^2}$$

$$\gamma_{max} = 2\sqrt{\left(\frac{2\epsilon_a-\epsilon_b-\epsilon_c}{3}\right)^2+\left(\frac{\epsilon_b-\epsilon_c}{\sqrt{3}}\right)^2}$$

$$\phi_1 = \frac{1}{2}\tan^{-1}\left[\frac{(\epsilon_b-\epsilon_c)/\sqrt{3}}{(2\epsilon_a-\epsilon_b-\epsilon_c)/3}\right]$$

$$\phi_2 = 90^{\circ}+\phi_1$$

Analytical Solutions

$$\sigma_{1,2} = E \left[\frac{\epsilon_a + \epsilon_b + \epsilon_c}{3(1-\nu)} \pm \frac{1}{(1+\nu)} \sqrt{\left(\frac{2\epsilon_a - \epsilon_b - \epsilon_c}{3}\right)^2 + \left(\frac{\epsilon_b - \epsilon_c}{\sqrt{3}}\right)^2} \right.$$

$$\tau_{max} = \frac{E}{1+\nu} \sqrt{\left(\frac{2\epsilon_a - \epsilon_b - \epsilon_c}{3}\right)^2 + \left(\frac{\epsilon_b - \epsilon_c}{\sqrt{3}}\right)^2}$$

Analytical Solutions

Example 4.3 *A delta rosette yields the following strain indications:*
$\varepsilon_a = -845\ \mu m/m$; $\varepsilon_b = 1220\ \mu m/m$ and $\varepsilon_c = 710\ \mu m/m$.

Calculate the maximum principal strain direction, the principal stresses and the maximum shear stress.

$$E = 200\ GPa,\ \nu = 0.285.$$

Solution. For a delta rosette

$$\varepsilon_x = \varepsilon_a = -845\ \mu m/m$$

$$\varepsilon_y = \frac{1}{3}(-\varepsilon_a + 2\,\varepsilon_b + 2\,\varepsilon_c)$$

$$= \frac{1}{3}(845 + 2440 + 1420)$$

$$= 1568.33\ \mu m/m$$

$$\gamma_{xy} = \frac{2}{\sqrt{3}}(\varepsilon_b - \varepsilon_c)$$

$$= \frac{2}{\sqrt{3}}(1220 - 770)$$

$$= 588.90\ \mu\ rad.$$

Analytical Solutions

The principal strains become

$$\varepsilon_{1,2} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) \pm \frac{1}{2}\sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2}$$

$$= \frac{1}{2}[(-845 + 1568.33)$$

$$\pm \sqrt{(-845 - 1568.33)^2 + (588.90)^2}]$$

$$= 1603.73\ \mu\,\text{m/m};\ -880.41\ \mu\text{m/m}$$

$$\gamma_{max} = \sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2}$$

$$= \sqrt{(-845 - 1568.33)^2 + (588.90)^2}$$

$$= 2484.14\ \mu\ \text{rad.}$$

Maximum principal strain direction is

$$\phi_1 = \frac{1}{2}\tan^{-1}\left(\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{588.90}{-845 - 1568.33}\right)$$

$$= \frac{1}{2}\tan^{-1}(-0.24402)$$

$$= -6.85^\circ = -(6^\circ - 51')$$

The principal stresses then become

$$\sigma_1 = \frac{E}{1-\nu^2}(\varepsilon_1 + \nu\,\varepsilon_2)$$

$$= \frac{200 \times 10^9}{1-(0.285)^2}(1603.75 - 0.285 \times 880.41)\,10^{-6}$$

$$= 294.482 \text{ MPa}$$

$$\sigma_2 = \frac{E}{1-\nu^2}(\varepsilon_2 - \nu\,\varepsilon_1)$$

$$= \frac{200 \times 10^9}{1-(0.285)^2}(-880.41 + 0.285 \times 1603.73)\,10^{-6}$$

$$= -92.154 \text{ MPa}$$

$$\tau_{max} = \frac{E}{2(1+\nu)}\gamma_{max}$$

$$= \frac{200 \times 10^9 \times 2484.14 \times 10^{-6}}{2(1+0.285)} = 193.32 \text{ MPa}$$

Analytical Solutions

(d) T – Delta Rosette

$$\phi_a = 0^\circ \Rightarrow \epsilon_a = \epsilon_x.$$

$$\phi_b = 60^\circ \Rightarrow \epsilon_b = \frac{1}{4}\left(\epsilon_x + 3\epsilon_y + \sqrt{3}\,\gamma_{xy}\right)$$

$$\phi_c = 120^\circ \Rightarrow \epsilon_c = \frac{1}{4}\left(\epsilon_x + 3\epsilon_y - \sqrt{3}\,\gamma_{xy}\right)$$

$$\phi_d = 90^\circ \Rightarrow \epsilon_d = \epsilon_y$$

$$\epsilon_x = \epsilon_a$$

$$\epsilon_y = \epsilon_d.$$

$$\gamma_{xy} = \frac{2}{\sqrt{3}}\left(\epsilon_b - \epsilon_c\right)$$

Analytical Solutions

$$\epsilon_{1,2} = \frac{1}{2}\left(\epsilon_a + \epsilon_d\right) \pm \frac{1}{2}\sqrt{\left(\epsilon_a - \epsilon_d\right)^2 + \left(\frac{2(\epsilon_b - \epsilon_c)}{\sqrt{3}}\right)^2}$$

$$\gamma_{max} = \sqrt{\left(\epsilon_a - \epsilon_d\right)^2 + \left(\frac{2(\epsilon_b - \epsilon_c)}{\sqrt{3}}\right)^2}$$

$$\sigma_{1,2} = \frac{E}{2}\left[\frac{\epsilon_a + \epsilon_d}{1 - \nu} \pm \frac{1}{1+\nu}\sqrt{\left(\epsilon_a - \epsilon_d\right)^2 + \left(\frac{2(\epsilon_b - \epsilon_c)}{\sqrt{3}}\right)^2}\right.$$

$$\tau_{max} = \frac{E}{2(1+\nu)}\sqrt{\left(\epsilon_a - \epsilon_d\right)^2 + \left(\frac{2(\epsilon_b - \epsilon_c)}{\sqrt{3}}\right)^2}$$

Analytical Solutions

Example 4.4 *The strain readings as measured by a T-delta rosette at a point in a stressed body are given by :* $\varepsilon_a = 225\ \mu\ m/m$, $\varepsilon_b = 305\ \mu\ m/m$, $\varepsilon_c = -294\ \mu\ m/m$, *and* $\varepsilon_d = -65\ \mu\ m/m$. *Determine the principal stresses, maximum principal stress direction and maximum shear stress.* $E = 200\ GPa$, $\nu = 0.30$.

Solution. For the *T*-delta rosette

$$\varepsilon_x = \varepsilon_a = 225\ \mu\ \text{m/m}$$

$$\varepsilon_y = \varepsilon_d = -65\ \mu\ \text{m/m}$$

$$\gamma_{xy} = \frac{2}{\sqrt{3}}(\varepsilon_b - \varepsilon_c)$$

$$= \frac{2}{\sqrt{3}}(305 + 274)$$

$$= 668.59\ \mu\ \text{rad.}$$

Analytical Solutions

Maximum principal stress direction

$$\phi_1 = \frac{1}{2}\tan^{-1}\left[\frac{\frac{2}{\sqrt{3}}(\varepsilon_b - \varepsilon_c)}{\varepsilon_a - \varepsilon_d}\right]$$

$$= \frac{1}{2}\tan^{-1}\frac{\frac{2}{\sqrt{3}}(305 + 274)}{225 + 65}$$

$$= \frac{1}{2}\tan^{-1}[2.30548]$$

$$= 33.27°$$

$$= 33° \, 16'$$

$$\sigma_{1,2} = \frac{E}{2}\left[\frac{\varepsilon_a + \varepsilon_d}{1 - \nu} \pm \frac{1}{1+\nu}\sqrt{(\varepsilon_a - \varepsilon_d)^2 + \frac{4}{3}(\varepsilon_b - \varepsilon_c)^2}\right]$$

$$= \frac{200 \times 10^9}{2}\left[\frac{225 - 65}{1 - 0.30} \pm \frac{1}{1 + 0.30}\sqrt{(225 + 65)^2 + \frac{4}{3}(305 + 274)^2}\right]$$

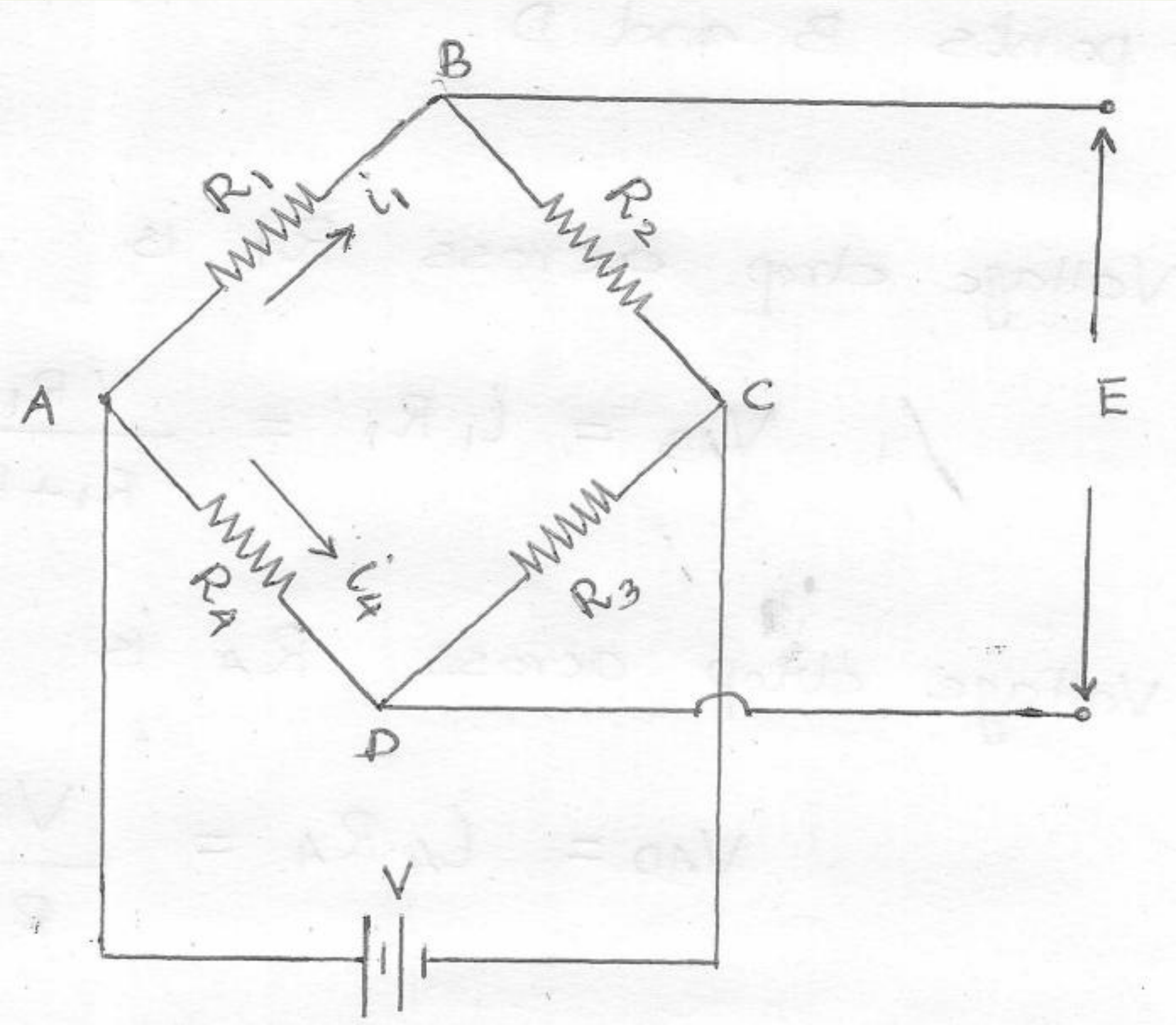
$$= 78.915 \text{ MPa}, \; -33.201 \text{ MPa}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{78.915 - 33.201}{2}$$

$$= 56.058 \text{ MPa}$$

Wheatstone Bridge Circuit

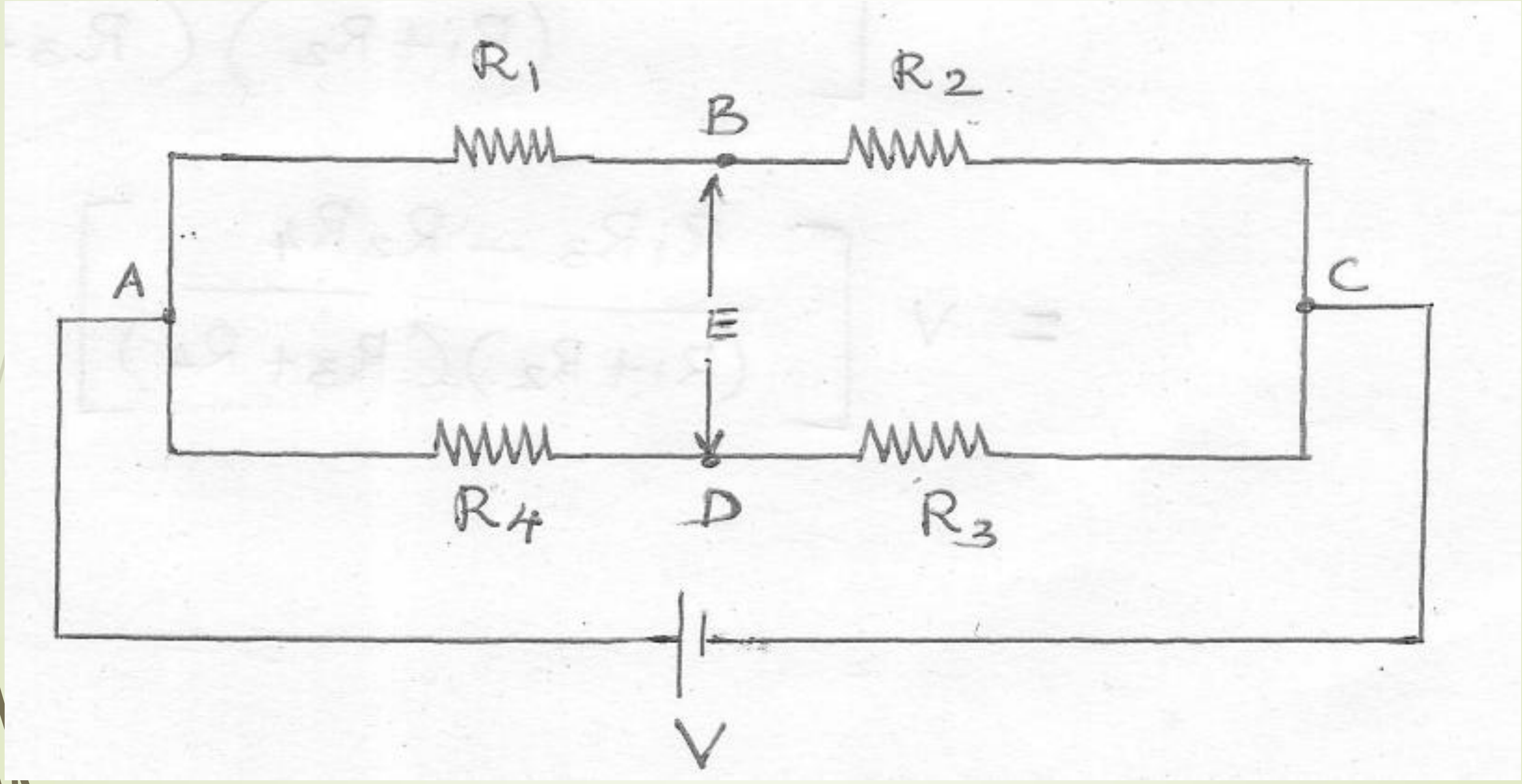
Wheatstone Bridge Circuit



Wheatstone Bridge Circuit

It Consists of four resistance arms with a battery and a meter. The resistances shown in each of the four arms can represent a Strain gauge. A voltage 'V' is applied to the bridge. Some measuring instrument such as a galvanometer is used to measure the output voltage.

Wheatstone Bridge Circuit



Wheatstone Bridge Circuit

Requirement for Balance

There is zero potential difference 'E' between the points B and D.

Voltage drop across R_1 is

$$V_{AB} = i_1 R_1 = \frac{V R_1}{R_1 + R_2}$$

Voltage drop across R_4 is

$$V_{AD} = i_4 R_4 = \frac{V R_4}{R_3 + R_4}$$

Wheatstone Bridge Circuit

The potential difference between B and D,

$$V_{BD} = V_{AB} - V_{AD} = E$$

$$\therefore E = V\left[\frac{R_1}{R_1+R_2} - \frac{R_4}{R_3+R_4}\right]$$

$$= V\left[\frac{R_1R_3 + R_1R_4 - R_1R_4 - R_2R_4}{(R_1+R_2)(R_3+R_4)}\right]$$

$$= V\left[\frac{R_1R_3 - R_2R_4}{(R_1+R_2)(R_3+R_4)}\right]$$

Wheatstone Bridge Circuit

For balanced bridge, $E = 0$

$$R_1 R_3 - R_2 R_4 = 0$$

$$R_1 R_3 = R_2 R_4$$

$$\boxed{\frac{R_1}{R_2} = \frac{R_4}{R_3}}$$

The ratio of resistances of any two adjascent arms of the bridge must be equal to the ratio of resistances of the remaining two arms taken in the same order.

Balanced Bridge

If R_1 is a strain gauge mounted on a specimen, the bridge can first balanced under no load by the altering the ratio $\frac{R_4}{R_3}$ suitably. After the specimen is loaded, the bridge can be balanced again by adjusting the ratio $\frac{R_4}{R_3}$. If the change in this ratio is known, then the change in the strain gauge resistance 'ΔR_1' due to load can be determined.

The corresponding strain, $\epsilon = \frac{\Delta R_1 / R_1}{F}$.

Wheatstone Bridge Circuit

Unbalanced Bridge

Consider an initially balanced bridge and then a change for ex: the resistance R_1 by an incremental amount ΔR_1. As the bridge then be unbalanced, a voltage ΔE will be produced between B and D.

Initially balanced, $\frac{R_1}{R_2} = \frac{R_4}{R_3}$

Wheatstone Bridge Circuit

When the bridge is unbalanced due to R_1 changing to $R_1 + \Delta R_1$,

The voltage, $$\Delta E = \frac{(R_1 + \Delta R_1) R_3 - R_2 R_4}{(R_1 + \Delta R_1 + R_2)(R_3 + R_4)} \cdot V$$

Wheatstone Bridge Circuit

$$\Delta E = \frac{R_1R_3 + \Delta R_1R_3 - R_2R_4}{R_1\left(1+\frac{\Delta R_1}{R_1}+\frac{R_2}{R_1}\right)R_3\left(1+\frac{R_4}{R_3}\right)} \cdot V$$

Dividing numerator and denominator by R_1R_3

$$\Delta E = \frac{1+\frac{\Delta R_1}{R_1} - \frac{R_2R_4}{R_1R_3}}{\left(1+\frac{\Delta R_1}{R_1}+\frac{R_2}{R_1}\right)\left(1+\frac{R_4}{R_3}\right)} \cdot V$$

Wheatstone Bridge Circuit

we have $R_2 R_4 = R_1 R_3$

$$\Delta E = \frac{\Delta R_1 / R_1}{\left(1 + \frac{\Delta R_1}{R_1} + \frac{R_2}{R_1}\right)\left(1 + \frac{R_4}{R_3}\right)} \cdot V$$

Substituting $\frac{R_2}{R_1} = \frac{R_3}{R_4} = m$

$$\Delta E = \frac{\Delta R_1 / R_1}{\left(1 + \frac{\Delta R_1}{R_1} + m\right)\left(1 + \frac{1}{m}\right)} \cdot V$$

Wheatstone Bridge Circuit

$$= \frac{\Delta R_1 / R_1}{(1+m)\left(1 + \frac{\Delta R_1/R_1}{1+m}\right)\left(\frac{1+m}{m}\right)} \cdot V$$

$$= \frac{m}{(1+m)^2} V \frac{\Delta R_1/R_1}{\left(1 + \frac{\Delta R_1/R_1}{1+m}\right)}$$

$$= \frac{mV}{(1+m)^2} \cdot \frac{\Delta R_1}{R_1} (1 - \eta)$$

Wheatstone Bridge Circuit

where $$(1-\eta) = \frac{1}{1 + \frac{\Delta R_1 / R_1}{1+m}}$$

$1-\eta$ is the non-linearity factor in the expression of ΔE.

Wheatstone Bridge Circuit

A general expression for ΔE can be derived in a similar manner for the case where all the four resistances R_1, R_2, R_3 and R_4 changed by incremental amounts ΔR_1, ΔR_2, ΔR_3 and ΔR_4 respectively.

The expression is given by,

$$\Delta E = \frac{m}{(1+m)^2} \cdot V \left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4} \right] \times (1-\eta)$$

Wheatstone Bridge Circuit

where,
$$1-\eta = \frac{1}{1+\frac{1}{1+m}\left(\frac{\Delta R_1}{R_1}+\frac{\Delta R_4}{R_4}+m\left(\frac{\Delta R_3}{R_3}+\frac{\Delta R_2}{R_2}\right)\right)}$$

The error introduced by the non-linearity of the wheatstone bridge circuit is negligibly small when the strain is measured by normal metallic strain gauges.

Wheatstone Bridge Circuit

Wheatstone Bridge Sensitivity

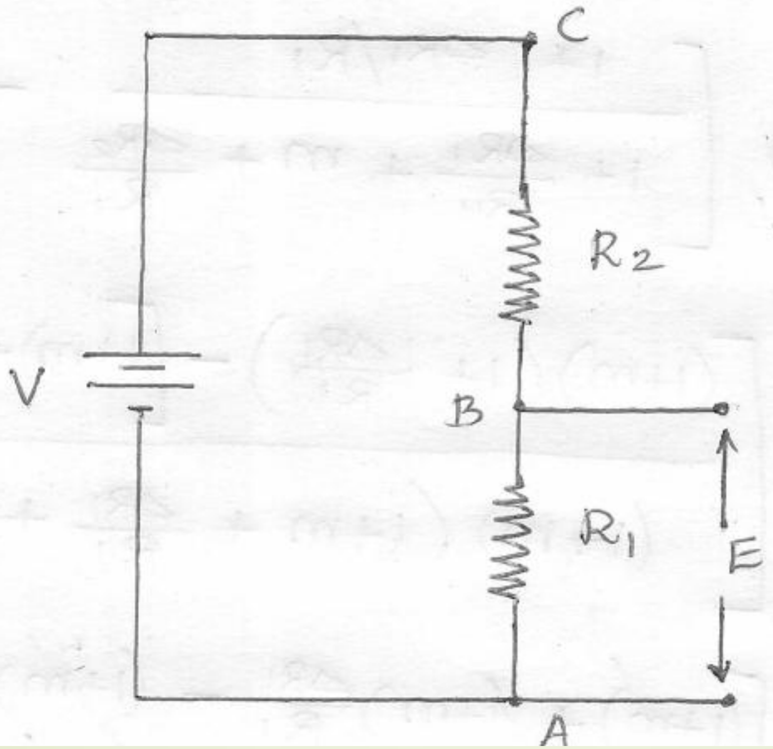
It is defined as the out of balance voltage ΔE produced by unit strain.

$$S_v = \frac{\Delta E}{\epsilon} = \frac{m}{(1+m)^2} \cdot \frac{V}{\epsilon} \left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4} \right]$$

Potentiometer Circuit

Potentiometer Circuit

The potentiometer circuit is well suited for dynamic strain measurements.



Potentiometer Circuit

Voltage across A and B,

$$E = V \cdot \frac{R_1}{R_1 + R_2}$$

$$= V \cdot \frac{1}{1 + \frac{R_2}{R_1}} = V \cdot \frac{1}{1+m}$$

where $m = \frac{R_2}{R_1}$

If R_1 and R_2 changed by incremental amounts ΔR_1 and ΔR_2 respectively,

Then, $$E + \Delta E = V \cdot \frac{R_1 + \Delta R_1}{R_1 + \Delta R_1 + R_2 + \Delta R_2}$$

Potentiometer Circuit

$$\therefore \Delta E = V\left[\frac{R_1+\Delta R_1}{R_1+\Delta R_1+R_2+\Delta R_2} - \frac{R_1}{R_1+R_2}\right]$$

$$= V\left[\frac{1+\Delta R_1/R_1}{1+\frac{\Delta R_1}{R_1}+\frac{R_2}{R_1}+\frac{\Delta R_2}{R_1}} - \frac{1}{1+m}\right]$$

$$= V\left[\frac{1+\Delta R_1/R_1}{1+\frac{\Delta R_1}{R_1}+m+\frac{\Delta R_2}{R_1}} - \frac{1}{1+m}\right]$$

Potentiometer Circuit

$$= V\left[\frac{(1+m)\left(1+\frac{\Delta R_1}{R_1}\right)-\left[(1+m)+\frac{\Delta R_1}{R_1}+\frac{\Delta R_2}{R_1}\right]}{(1+m)\left(1+m+\frac{\Delta R_1}{R_1}+\frac{\Delta R_2}{R_1}\right)}\right]$$

$$= V\left[\frac{(1+m)+(1+m)\frac{\Delta R_1}{R_1}-(1+m)-\frac{\Delta R_1}{R_1}-\frac{\Delta R_2}{R_1}}{(1+m)\left(1+m+\frac{\Delta R_1}{R_1}+\frac{\Delta R_2}{R_1}\right)}\right]$$

$$= V\left[\frac{\frac{\Delta R_1}{R_1}\left(1+m-1\right)-\frac{\Delta R_2}{R_1}}{(1+m)\left(1+m+\frac{\Delta R_1}{R_1}+\frac{\Delta R_2}{R_1}\right)}\right]$$

Potentiometer Circuit

$$= V\left[\frac{m\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_1} \times \left(\frac{R_2}{R_2}\right)}{(1+m)\left(1+m + \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_1}\right)}\right]$$

$$= \frac{V \cdot m\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2}\right)}{(1+m)\left(1+m + \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_1}\right)}$$

Potentiometer Circuit

$$= \frac{V m \left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} \right)}{(1+m)^2 + (1+m)\left[\frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_1} \cdot \frac{R_2}{R_2} \right]}$$

$$= \frac{V m \left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} \right)}{(1+m)^2 + (1+m)\left[\frac{\Delta R_1}{R_1} + m \cdot \frac{\Delta R_2}{R_2} \right]}$$

$$= \frac{V m}{(1+m)^2} \left[\frac{\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} \right)}{1 + \frac{1}{1+m} \left(\frac{\Delta R_1}{R_1} + m \frac{\Delta R_2}{R_2} \right)} \right]$$

Potentiometer Circuit

$$= \frac{V m}{(1+m)^2} \left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} \right] (1-\eta)$$

where $$(1-\eta) = \frac{1}{1 + \frac{1}{1+m}\left(\frac{\Delta R_1}{R_1} + m \frac{\Delta R_2}{R_2} \right)}$$ is

the non-linear term. It may be neglected.

Potentiometer Circuit

Circuit sensitivity, $S_v = \frac{\Delta E}{E}$

$$= \frac{m}{(1+m)^2} \cdot \frac{V}{E} \left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} \right]$$

Potentiometer Circuit

Potentiometer Circuit

MODULE 4

Theory of Light

Theory of Light

Characteristics

Frequency and Colour-Number of oscillations per second. Generally denoted in Hertz or in radians per second.

Velocity- velocity of light waves in vacuum is $3x10^8$ m/s. The ratio of velocity of light in air to the velocity of light in the medium is called refractive index of the medium.

Wavelength-Distance travelled during one complete vibration. It is usually measured in angstroms. $1A^0=10^{-10}$m

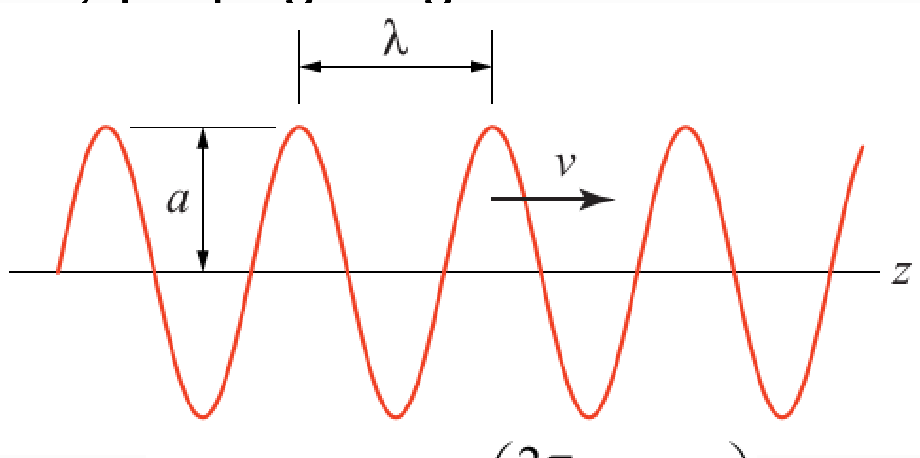
Amplitude-Magnitude of the disturbance.

Phase-stage of the cycle reached at that instant.

Theory of Light

Wave theory of light

- The theory of photoelasticity is based on the wave nature of light.
- Light is regarded as a sinusoidal electromagnetic wave having transverse amplitude ‘a’ and longitudinal wavelength ‘λ’, propagating in the z direction with velocity v.



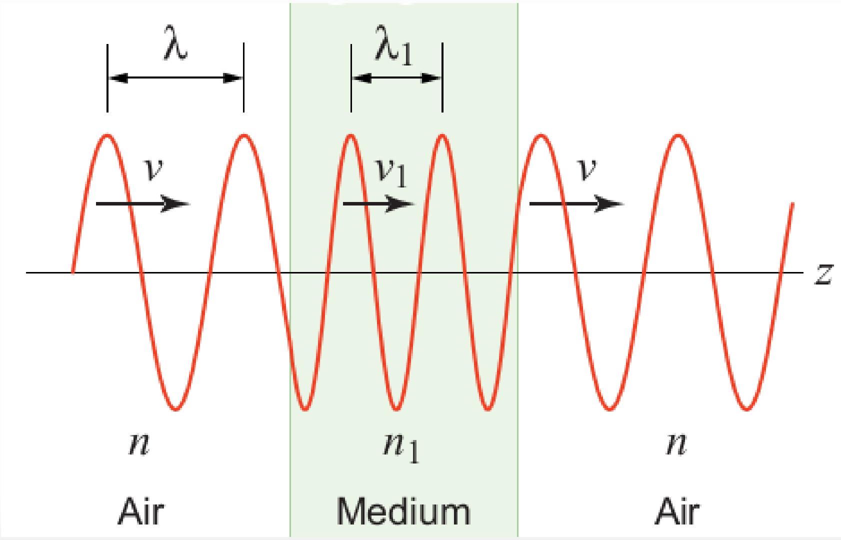
$$a\cos\Phi = a\cos\left(\frac{2\pi}{\lambda}(z - vt)\right)$$

Theory of Light

Refraction

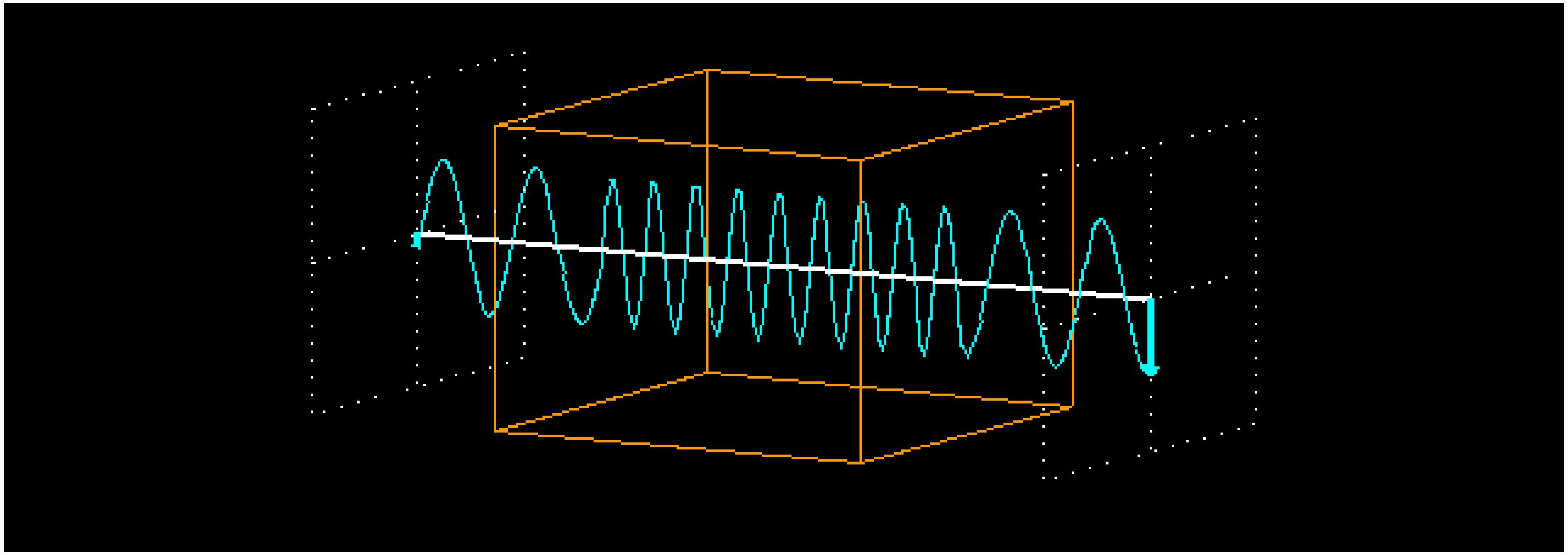
- When light passes through any medium, its velocity decreases to a value $$v_1 = \frac{v}{n_1}$$

  where n1 denotes the refractive index of the medium.



Theory of Light

Refraction



Theory of Light

Refraction

- However, the frequency *f* of the wave is unaffected. Therefore the wavelength λ1 must also decrease proportionally:

$$\lambda_1 = \frac{v_1}{f} = \frac{v}{n_1 f} = \frac{\lambda}{n_1}.$$

- Note that the time t1 required for light to propagate through a thickness h of medium1 having index of refraction n1 is

$$t_1 = \frac{h}{v_1} = n_1 \frac{h}{v}.$$

Theory of Light

Double Refraction

- If similar light waves pass through the same thickness h of two media having indices of refraction n_1 and n_2 and if $n_2 > n_1$ then the difference in transit times $t_2 - t_1$ will be,

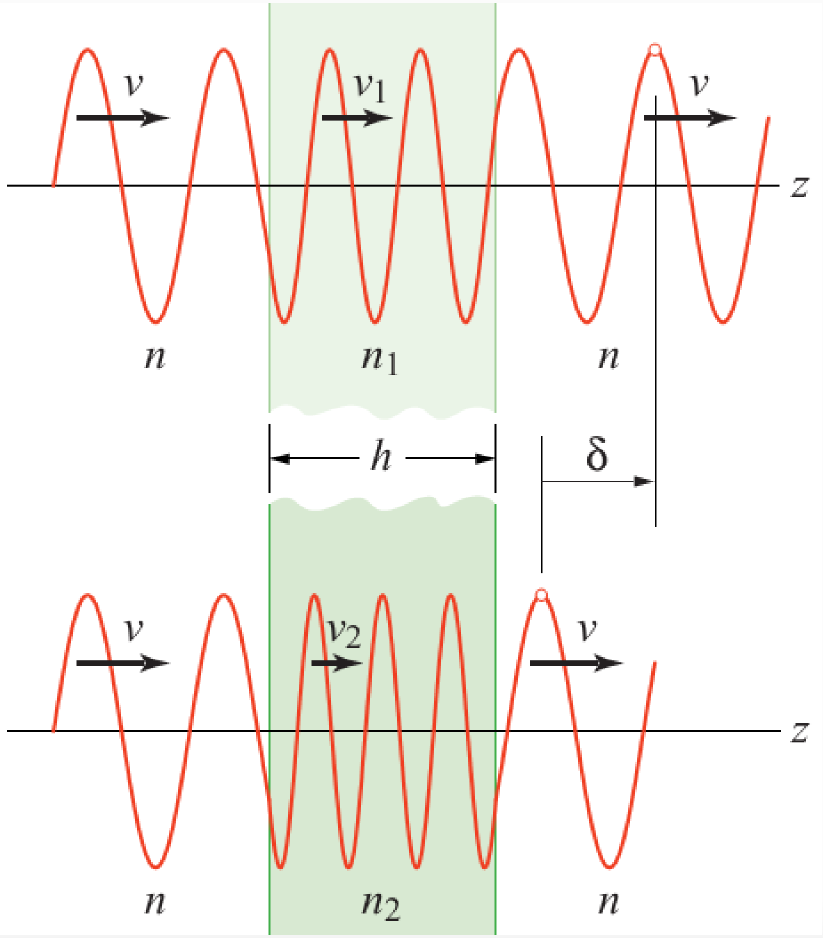
$$t_2 - t_1 = \frac{h}{v_2} - \frac{h}{v_1} = \frac{h}{v}(n_2 - n_1)$$

- Therefore, the phase difference δ between the two waves after they emerge from the media will be

$$\delta = v(t_2 - t_1) = h(n_2 - n_1)$$

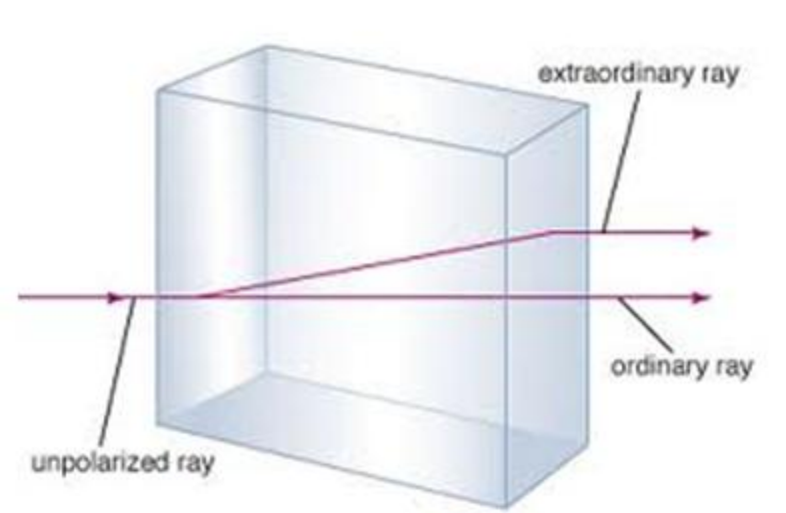
Theory of Light

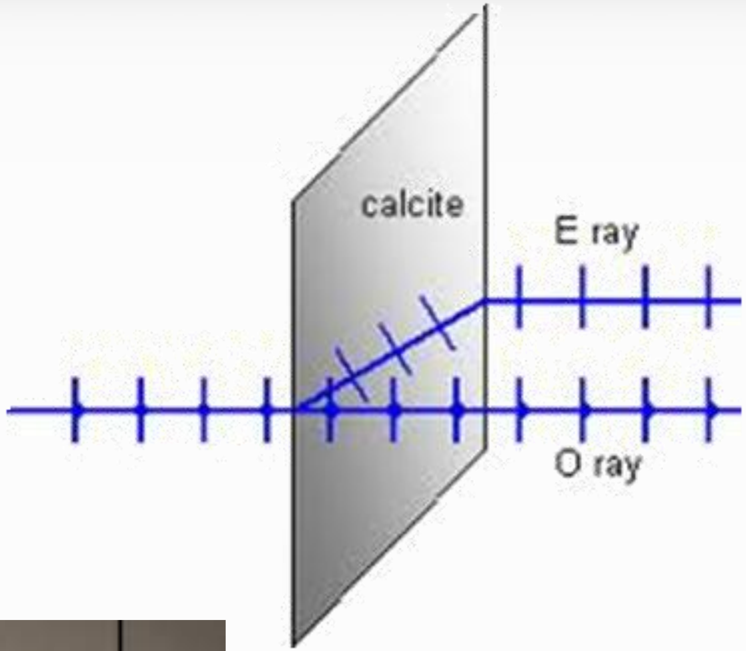
Double Refraction



Theory of Light

Double Refraction





Theory of Light

Polarization

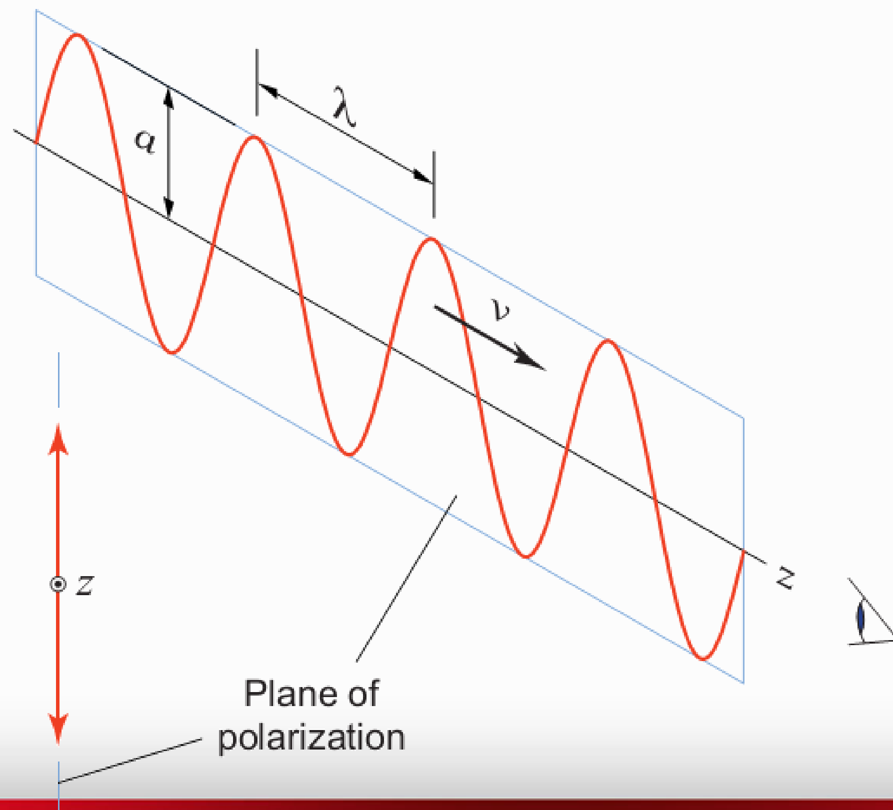
- A given light wave has an amplitude vector that is always perpendicular to its propagation direction.

- However, for ordinary light, the orientation of the amplitude vector in the plane perpendicular to the propagation direction is totally random.

Theory of Light

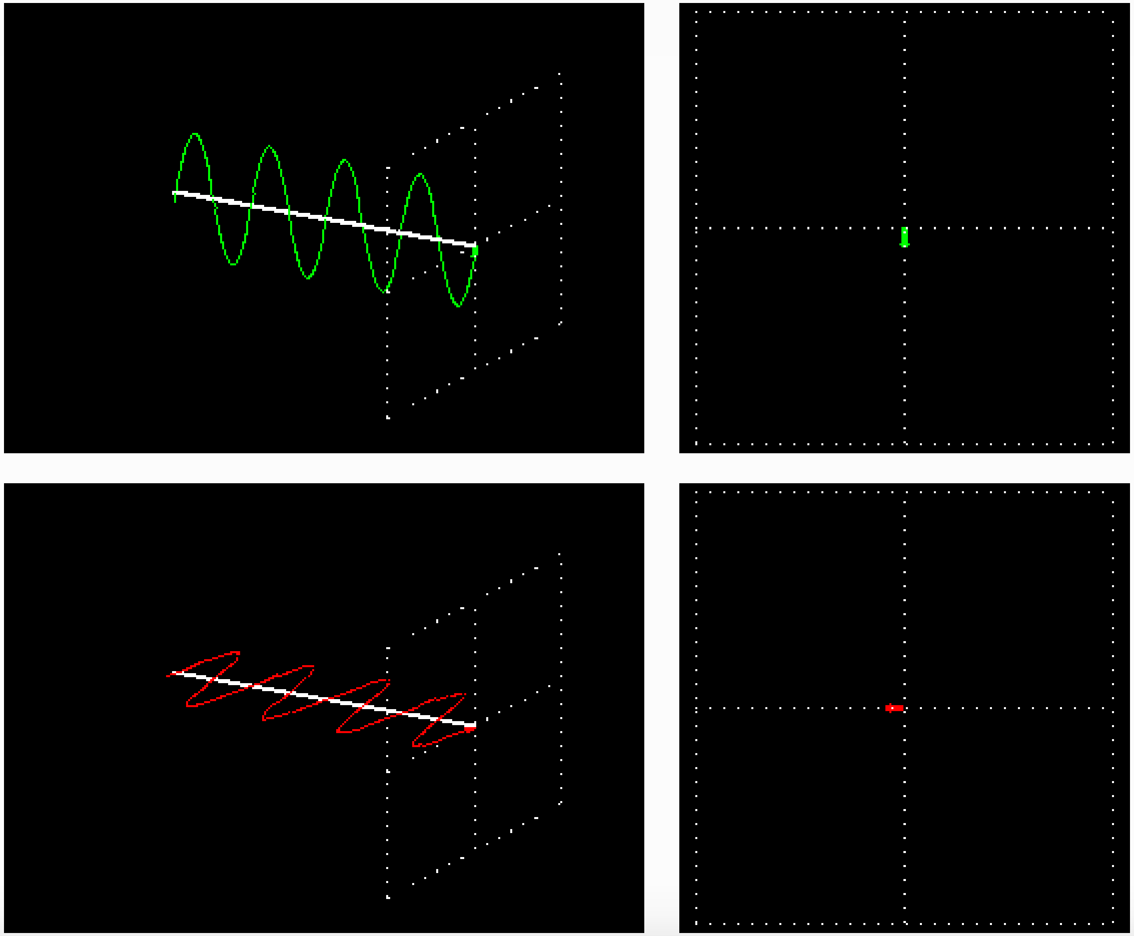
Plane-polarized light

- If the amplitude vectors of all light rays emanating from a source are restricted to a single plane, the light is said to be plane polarized.



Theory of Light

Plane-polarized light



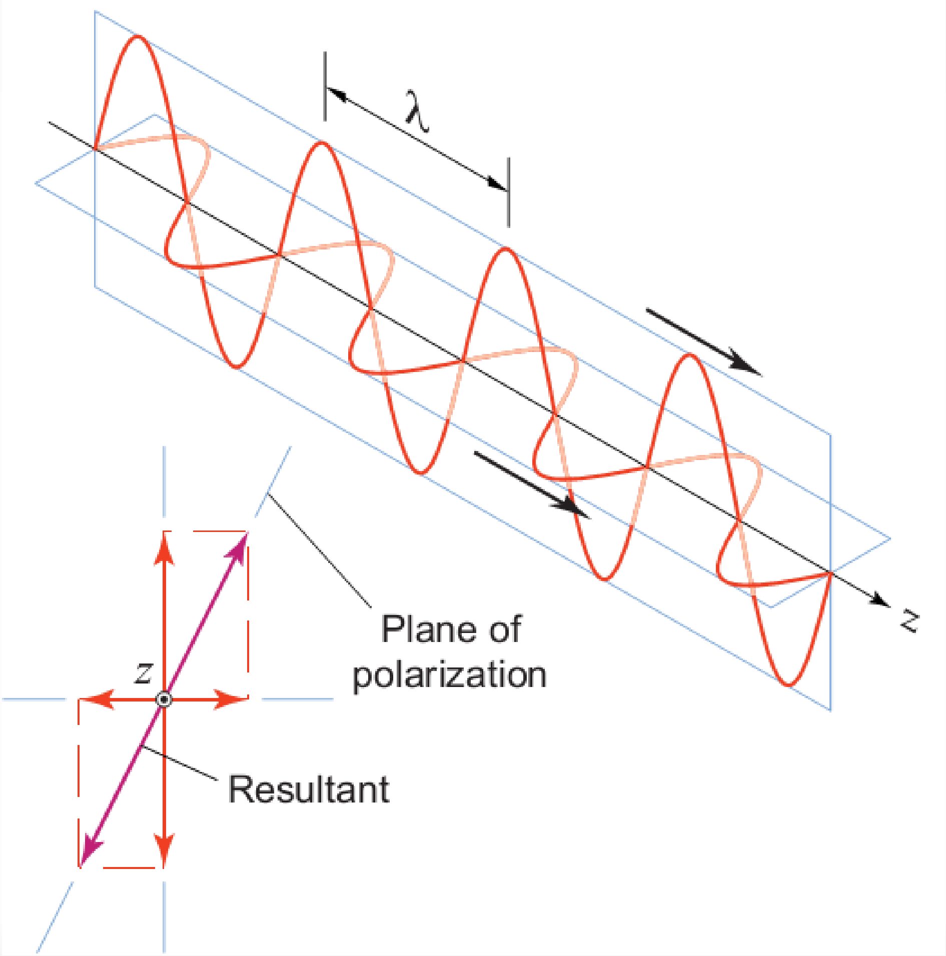
Theory of Light

Addition of two plane-polarized waves in phase

- Consider the addition of two plane-polarized waves that are in phase, but that have different planes of polarization, produces a new plane-polarized wave having the same frequency, wavelength, and phase as the component waves.

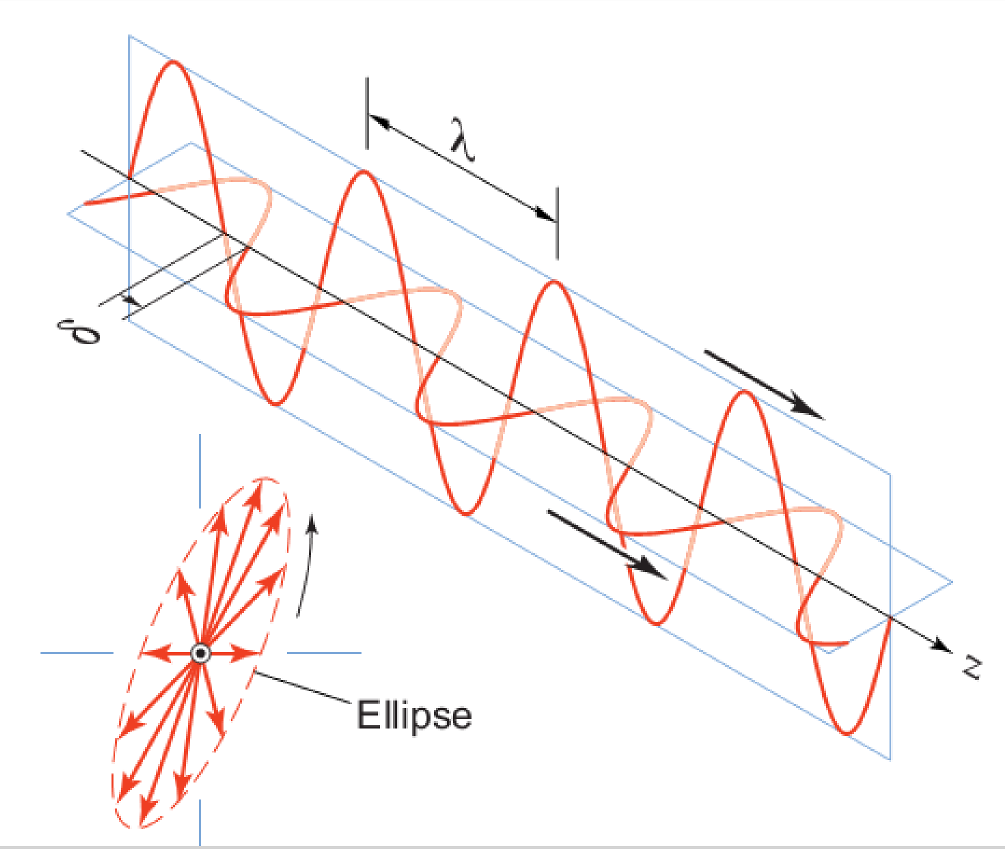
Theory of Light

Addition of two plane-polarized waves in phase



Elliptically polarized light

- When two plane-polarized waves of arbitrary amplitude and different phase are combined elliptically polarized light results.



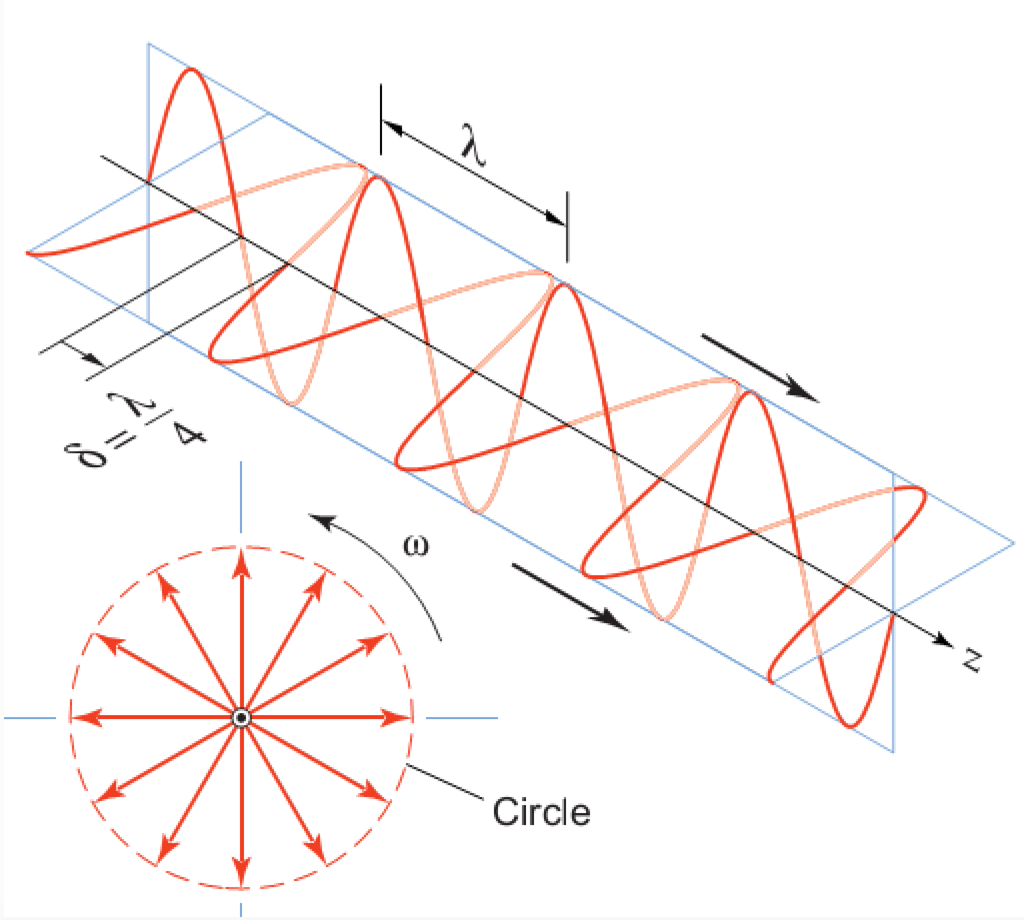
Theory of Light

Circularly polarized light

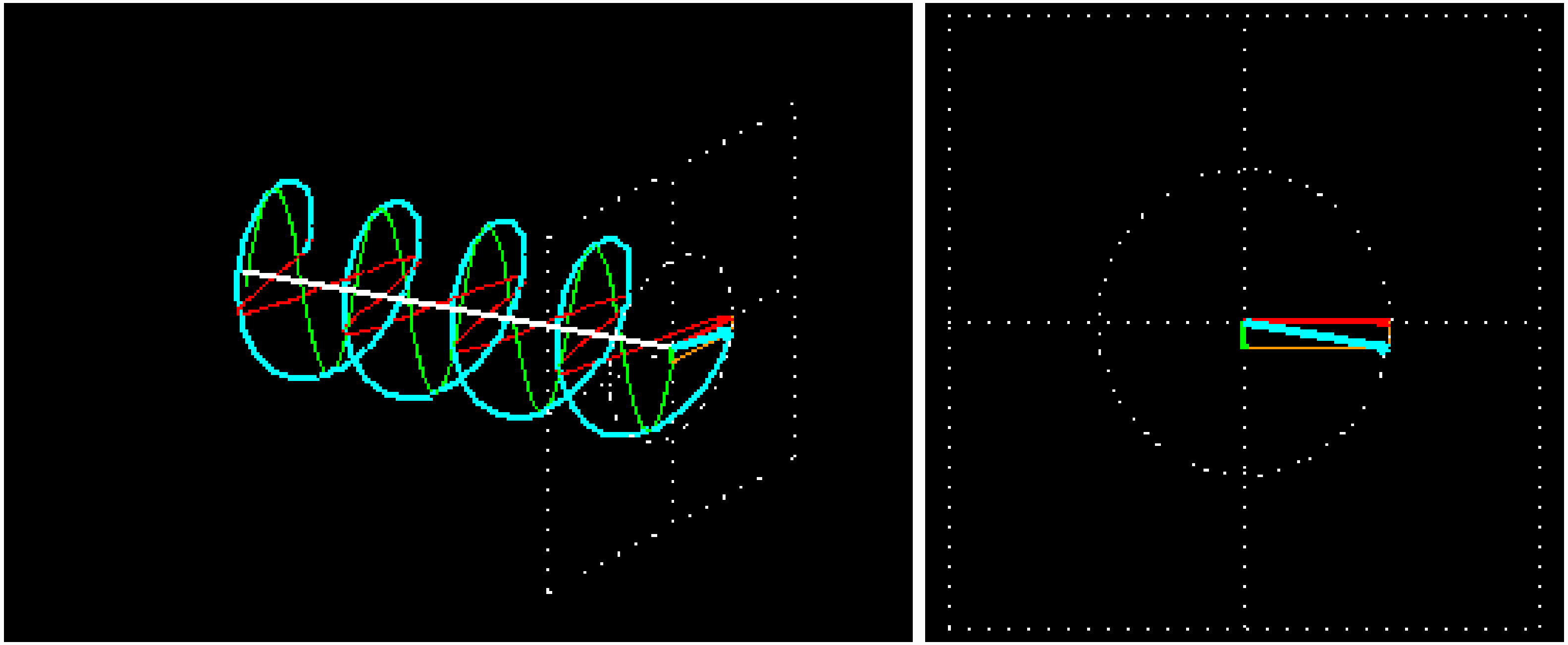
- A very important special case of elliptically polarized light is circularly polarized light, which can be created by combining orthogonal plane-polarized waves of equal amplitude that are out of phase by exactly one-quarter of a wavelength, i.e. $\delta = \lambda/4$.

Theory of Light

Circularly polarized light



Circularly polarized light

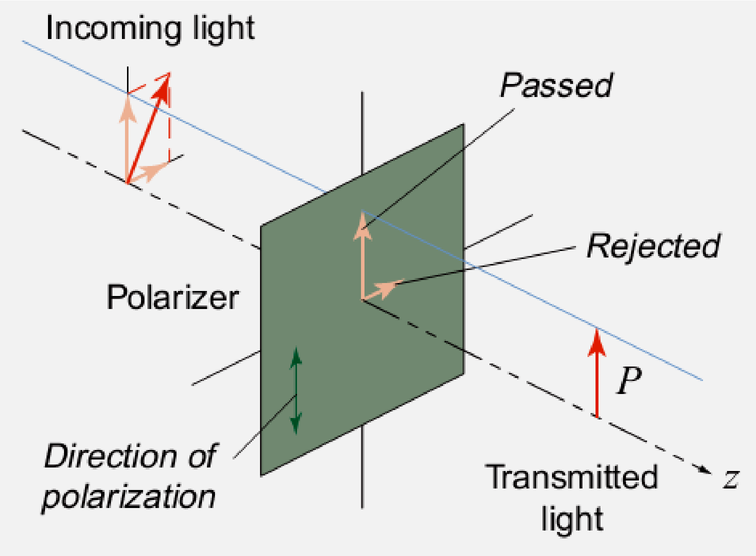


Optical elements

The method of photoelasticity requires the use of two types of optical element—the polarizer and the wave plate.

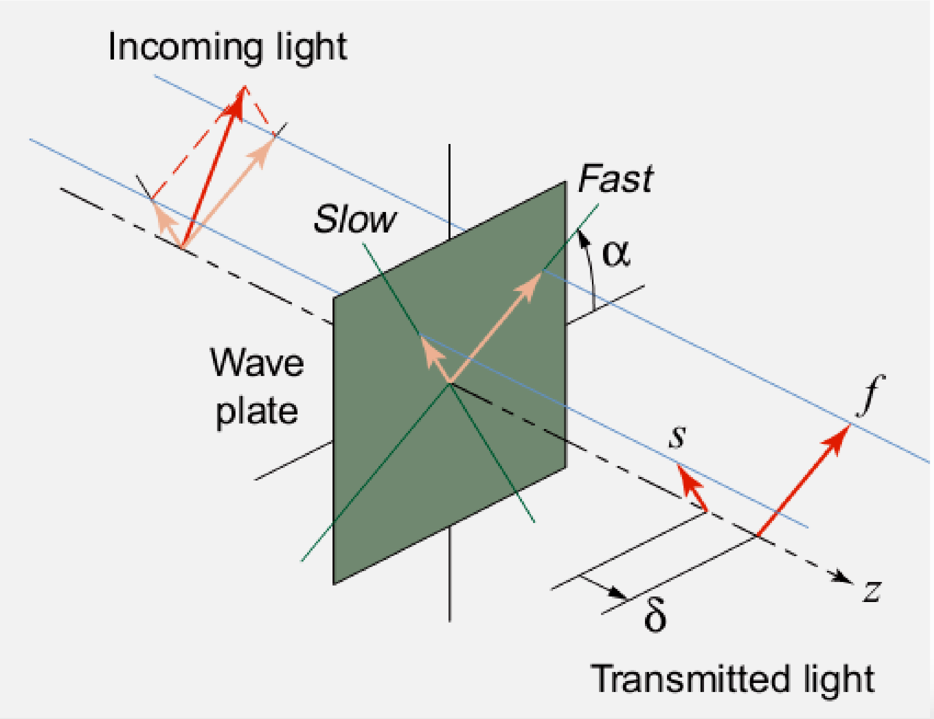
Polarizer

- A polarizer is an element that converts unpolarized light into plane-polarized light.



Wave plate

- A wave plate resolves incident light into two components, but instead of rejecting one of these components, it retards it relative to the other component.



Theory of Light

Wave plate

- In the figure, the "fast" axis of the wave plate makes an angle α with respect to an arbitrarily chosen reference direction.

- The component **f** of the incident light with amplitude vector in this orientation is retarded somewhat as it passes through the wave plate. However, the orthogonal component **s** is retarded even more, resulting in a phase lag δ between this "slow" component and the "fast" one.

- The term double refraction is often used to describe this behavior.

Theory of Light

- Wave plates may be either permanent or temporary. A permanent wave plate has a fixed fast-axis orientation α and a fixed relative retardation δ.

- A temporary wave plate has the ability to produce double refraction in response to mechanical stimulus.

- Photoelastic specimens are temporary wave plates.

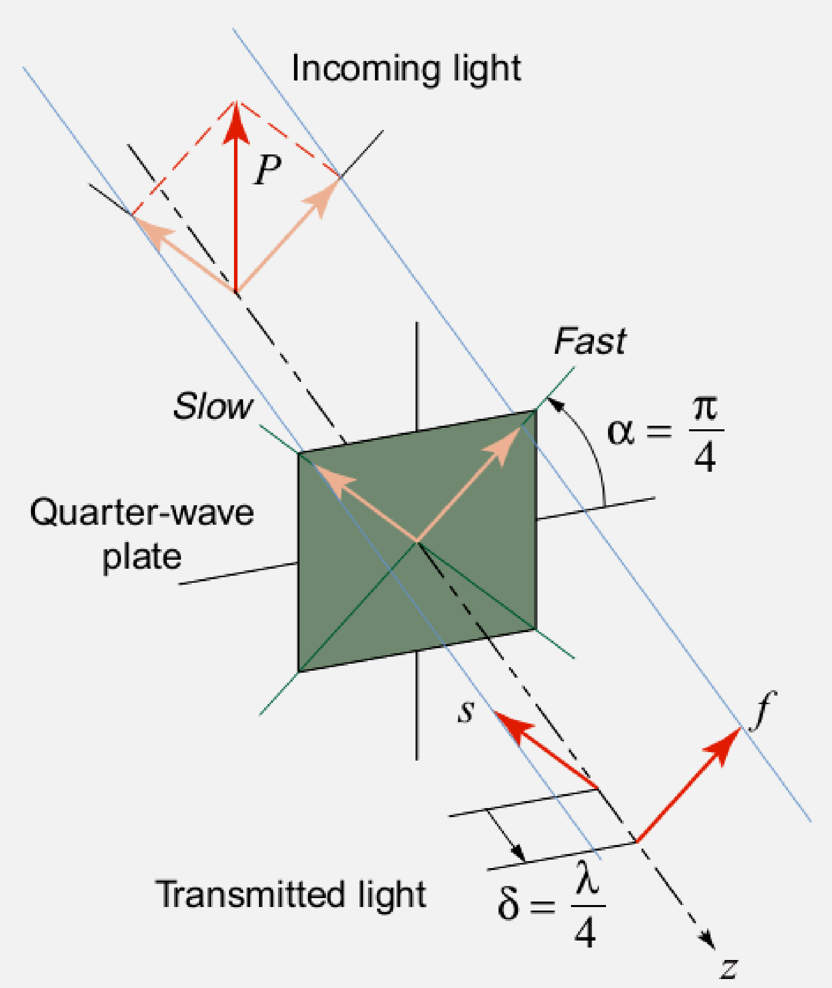
Theory of Light

Quarter-wave plate

- A quarter-wave plate is a permanent wave plate that induces a phase shift δ equal to λ/ 4, where λ is the wavelength of the light being used.

- The quarter-wave plate is an essential element in a circular polariscope.

Quarter-wave plate



Photoelasticity

- Photoelasticity is a nondestructive, whole-field, graphic stress-analysis technique based on an opto-mechanical property called birefringence (double refraction), possessed by many transparent polymers.

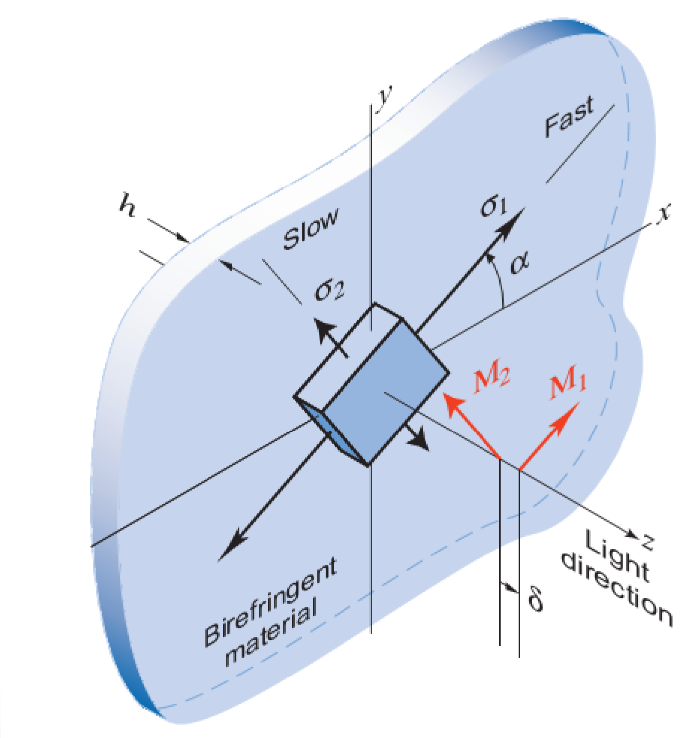
Induced Birefringence

- Photoelastic materials are birefringent, that is, they act as temporary wave plates, refracting light differently for different light-amplitude orientations, depending upon the state of stress in the material.

- In the unloaded state, the material exhibits an index of refraction 'n_0' that is independent of orientation. Therefore, light of all orientations propagating along all axes through the material propagate with the same speed, namely v/n_0.

Induced Birefringence

- In the loaded state, the orientation of a given light amplitude vector with respect to the principal stress axes, and the magnitudes of the principal stresses, determine the index of refraction for that light wave.



Stress Optic Law

- Maxwell reported that the changes in the index of refraction were linearly proportional to stresses and followed by the relationship,

$$n_1 - n_0 = c_1\sigma_1 + c_2\sigma_2$$

$$n_2 - n_0 = c_1\sigma_2 + c_2\sigma_1$$

Where

c_1 = direct stress-optic coefficient

c_2 = transverse stress-optic coefficient

n_0 = refractive index of material in unstressed state

Stress Optic Law

- Let us suppose that light waves with amplitude vector in the σ2 direction propagate more slowly through the material than those with amplitude vector in the σ1 direction. Then,

$$n_2 > n_1$$

- From the above equations,

$$\begin{aligned} n_2 - n_1 &= (c_2 - c_1)(\sigma_1 - \sigma_2) \\ &= c(\sigma_1 - \sigma_2), \end{aligned}$$

- where c is called the relative or differential stress-optic coefficient, which is a material property and is expressed in brewster. $(1\,\text{brewster} = 10^{-12}\ \text{m}^2/\text{N})$

Stress Optic Law

- Now consider the emerging phase difference δ between orthogonal components M1 and M2 of a light wave that entered the material from the back in phase and that were aligned in the principal stress directions.

$$\begin{aligned} \delta &= h(n_2 - n_1) \\ &= hc(\sigma_1 - \sigma_2) \end{aligned}$$

where h is the thickness of the material in the light-propagation direction.

Stress Optic Law

- It is often written in terms of the number N of complete cycles of relative retardation, or, equivalently, in terms of the angular phase difference Δ, as follows:

$$N = \frac{\Delta}{2\pi} = \frac{\delta}{\lambda} = \frac{hc(\sigma_1 - \sigma_2)}{\lambda} = h\frac{\sigma_1 - \sigma_2}{f_\sigma}$$

where f_σ is called the material fringe value. $f_\sigma = \frac{\lambda}{c}$

Model fringe value, $F = \frac{f_\sigma}{h} = \frac{\lambda}{ch}$

- One wavelength of relative path difference or a relative phase difference of 2π is called a fringe.

Stress Optic Law

In summary, a birefringent material does two things:

- ❑ It resolves the incoming light into 2 components—one parallel to σ1 and the other parallel to σ2, and

- ❑ It retards one of the components, M2, with respect to the other, M1 , by an amount δ that is proportional to the principal stress difference σ1- σ2.

Transmission Photoelasticity

- Certain non-crystalline transparent materials, notably some polymeric plastics, are optically isotropic under normal conditions but become doubly refractive or birefringent when stressed.

- This effect normally persists while the loads are maintained but vanishes almost instantaneously or after some interval of time depending on the material and conditions of loading when the loads are removed.

Polariscopes

- A polariscope is an optical setup that allows the birefringence in specimens to be analyzed.
- It consists of a light source, a polarizer, an optional quarter-wave plate, a specimen, another optional quarter-wave plate, and a second polarizer called the analyzer.

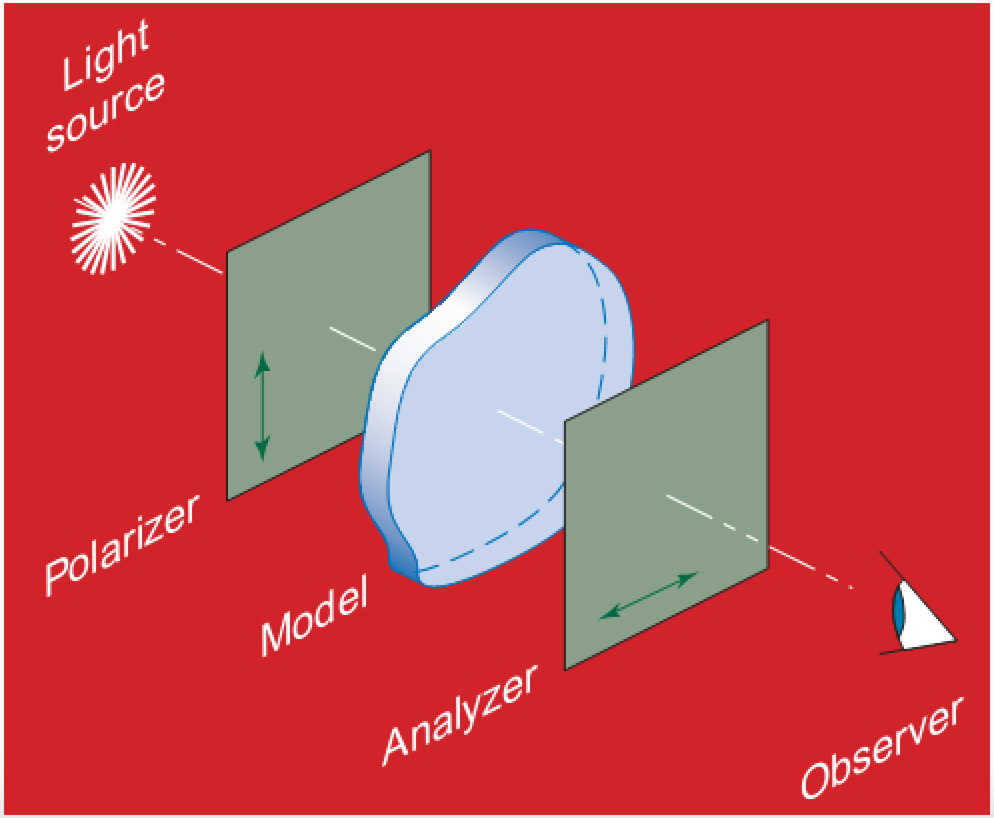
- Two types of polariscope are commonly employed

  a) Plane Polariscope

  b) Circular Polariscope.

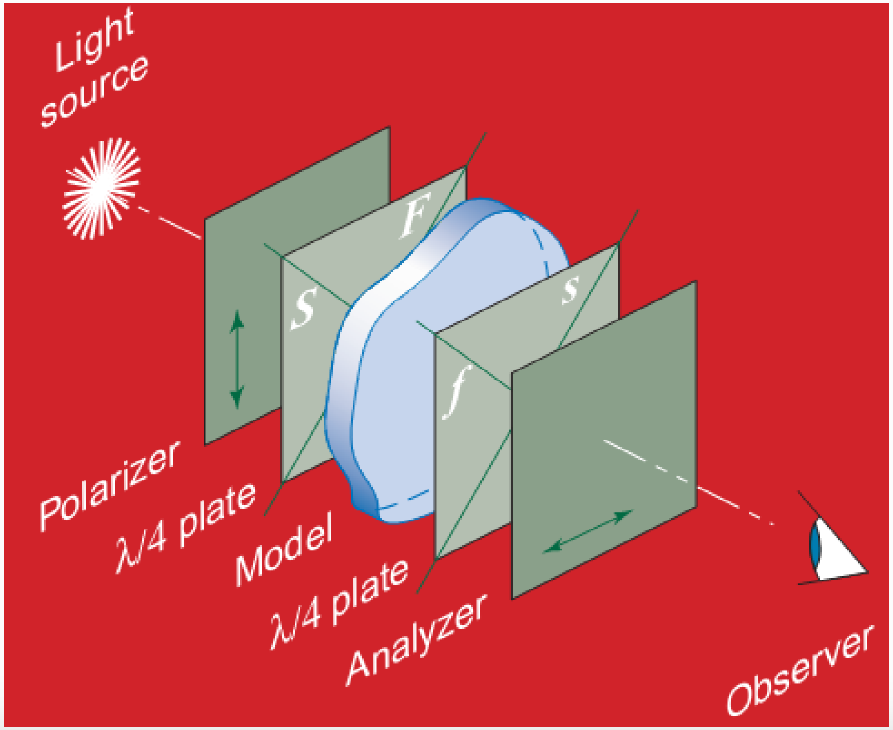
Plane Polariscope

- The plane polariscope consists of a light source, a polarizer, the specimen, and an analyzer that is always crossed with respect to the polarizer.



Circular Polariscope

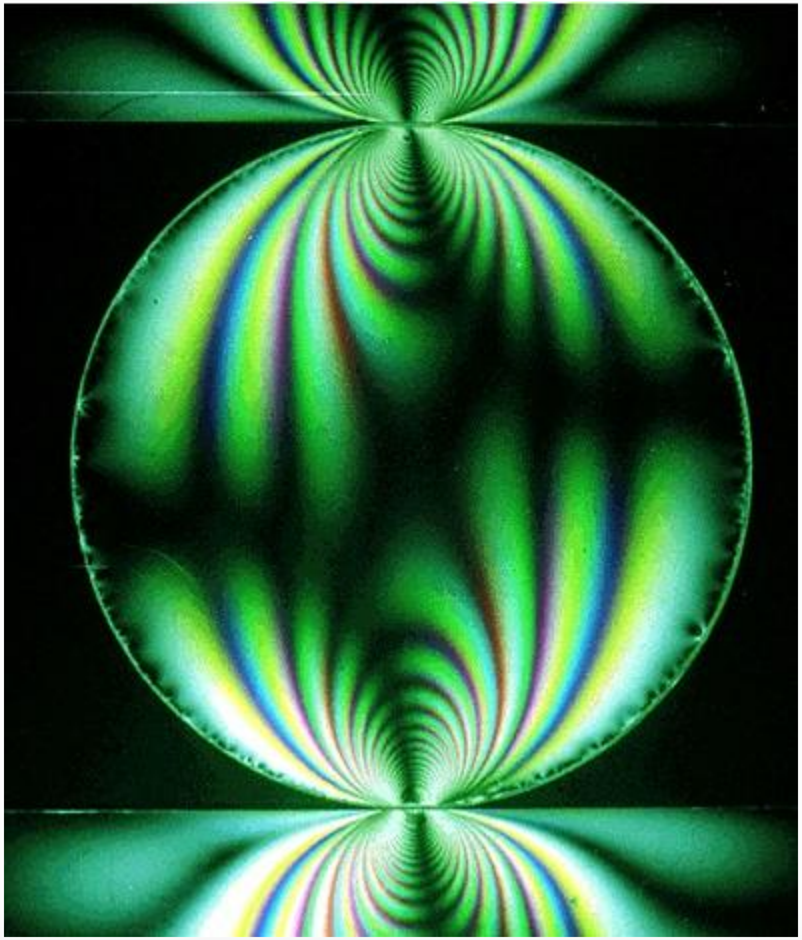
- The circular polariscope consists of a light source, a polarizer, a quarter-wave plate oriented at 45º with respect to the polarizer, the specimen, a second quarter-wave plate, and an analyzer.



Circular Polariscope

- The two quarter-wave plates are generally crossed (as shown in the figure) to minimize error due to imperfect quarter-wave plates. The analyzer is either crossed with respect to the polarizer or parallel to the polarizer.

Fringe Patterns On a Diametrally Loaded Circular Disk



Reflection Photoelasticity

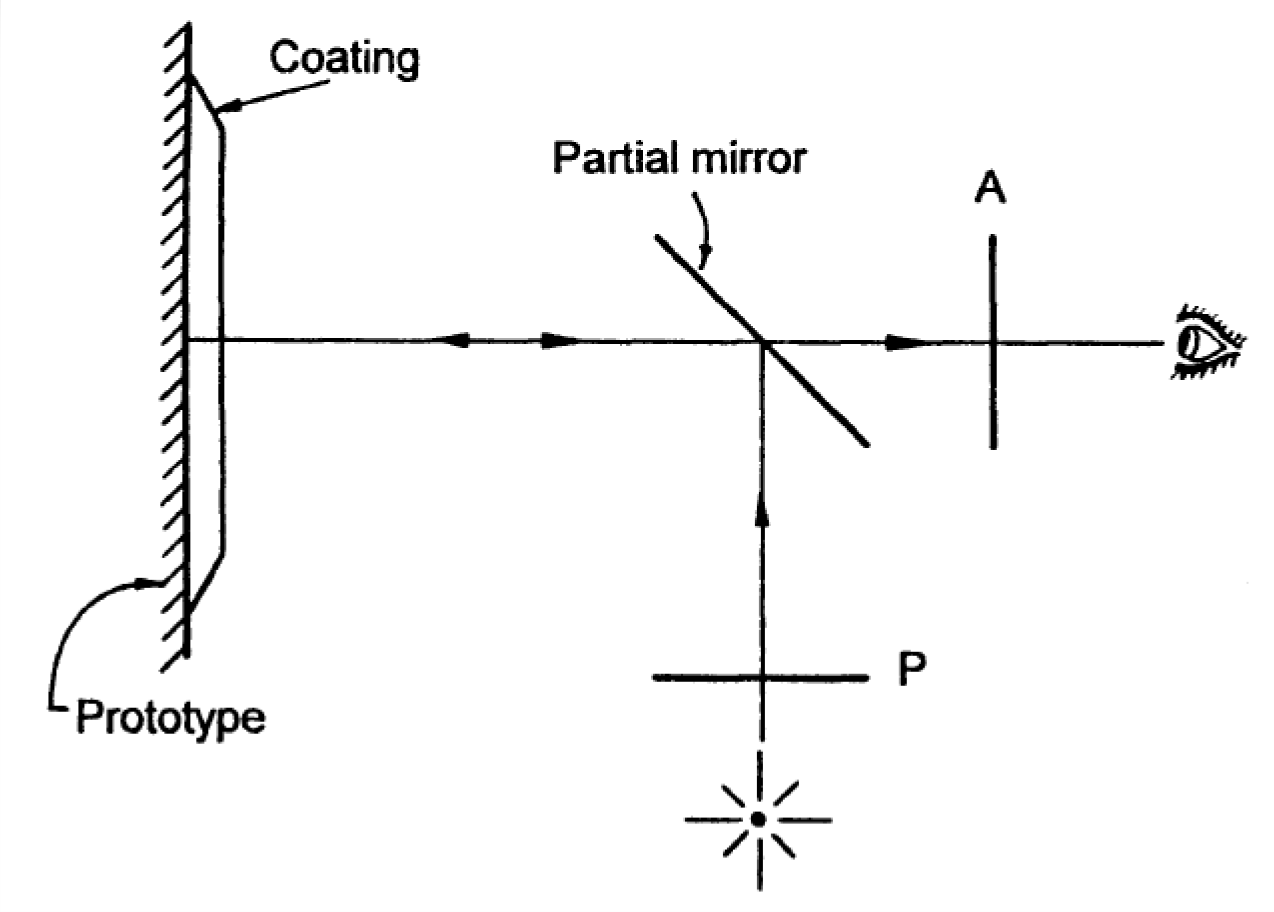
- Reflection photoelasticity is an extension of transmission photoelastic analysis for the analysis of opaque prototypes.

- A thin temporarily birefringent coating is pasted on the prototype with a reflective backing at the interface.

- The prototype is loaded appropriately and a reflection polariscope is used for collecting the optical information.

- Unlike in transmission photoelasticity, the birefringent plastic is not directly loaded but the surface strains developed on the prototype are transmitted through the adhesive to the coating.

Reflection Photoelasticity



Optical arrangement to obtain normal incidence in a reflection polariscope

PHOTOELASTIC MATERIALS

Photoelastic Materials

Selection of Photoelastic Materials

An ideal photoelastic material should have the following properties

1. Transparent to light used in the polariscope.
2. Easily machinable by conventional means.
3. It should have high optical sensitivity as indicated by low fringe values
4. It should have linear characteristics with respect to stress-strain, stress-fringe order and strain-fringe order properties for model to prototype scaling.
5. It should be free from residual stresses.
6. It should have both mechanical and optical isotropy and homogeneity.
7. There should be absence of undue optical and mechanical creep.
8. It should have high modulus of elasticity, ultimate strength and hardness to avoid distortion and contact problems.
9. It should be free from time-edge effects.
10. The material fringe values f_σ or f_ε should remain constant during moderate temperature changes.
11. It should have moderate cost.
12. It should have ˙ igh rigidity.

Photoelastic Materials

1. **Epoxy Resins.** These are condensation products of epichlorohydrin and polyhydric phenol. Although they are available as a full polymerized material, usually in the form of flat sheets suitable for two-dimensional photoelastic work, epoxies are now-a-days supplied as a basic resin and hardener or curing agent. The finished resin is formed by the chemical reaction of basic resin and hardener.

In India the most commonly used epoxy resins are Araldite CY-230 and Hardener HY-951 (both liquids), which are of the cold-setting type and Araldite CT-200 (solid) and Hardener HT-901, which are of the hot setting type, and manufactured by Hindustan CIBA-Geigy Limited.

Epoxy resins possess good optical and mechanical properties, slightly susceptible to time edge effects and are available at comparatively low cost. They can be easily cast, machined, finished and cemented for making models of complicated shapes.

Photoelastic Materials

2. **Columbia Resin CR-39.** This is allyl diglycol carbonate which is produced by reacting phosgene with diethylene glycol to obtain a chloroformate, which is then esterified with allyl alcohol to yield a monomer. The monomer is polymerized by heating in the presence of catalyst (benzoyl peroxide) and the resultant sheet is a crystal-clear product. It is available in large sheets. It is brittle and cannot be machined but can be easily routed. It is less prone to time edge effect but suffers to some extent from creep. It has relatively low fringe value and sensitivity index.

3. **Castolite or Homolite 100.** It is a polyester resin which can be cast into sheet form between glass plates by adding a catalyst to the monomer and curing at about 93°C to complete polymerisation. The surfaces of the commercially available sheets are of optical quality, and the material is free of residual stresses. Models can be prepared by routing only. It is insensitive to time edge effect and is best suited for prolonged testing. The material exhibits both a low figure of merit and sensitivity index.

Photoelastic Materials

4. **Polymethacrylate.** It is marketed under the trade name of Plexiglass (Germany and USA) and Perspex (U.K.). It is available in the form of flat sheets and round bars in a wide variety of sizes. The material is highly transparent, colourless and free from initial stress. It exhibits practically no creep or time edge effect. The optical sensitivity is very low.

5. **Bakelite (Catalin 61-893).** It is glyptal resin which is glycerine phthallic anhydride. Figure of merit and sensitivity index are quite high for bakelite. It is water clear, machinable with ordinary tools, not too brittle yet hard enough, isotropic, creeps moderately, unavailable in large sheets and exhibits time edge effect. It is now very rarely used for photoelastic work.

Photoelastic Materials

6. **Polycarbonate.** It is marketed under the trade name Makrolon (Europe) and Lexan (U.S.A.). It has very high optical sensitivity and figure of merit. It is free from time-edge effect and shows little creep at room temperature. Polycarbonate is a thermoplastic and is available in sheet form. Residual stresses produced during manufacturing may be removed by careful heat treatment. It can be routed by using water as a coolant.

7. **Polyurethane Rubber.** It is a rubber like plastic of very low elastic modulus and very high optical sensitivity. It is a highly transparent, amber coloured material and is practically free from time-edge effect. Optical and mechanical creep are negligible at room temperature. It is mainly used for demonstration purposes. It is available under the trade-name Photoflex (U.K.) and Hysol 4485 (U.S.A.). It is a popular material for dynamic photoelasticity and study of body force problems.

Photoelastic Materials

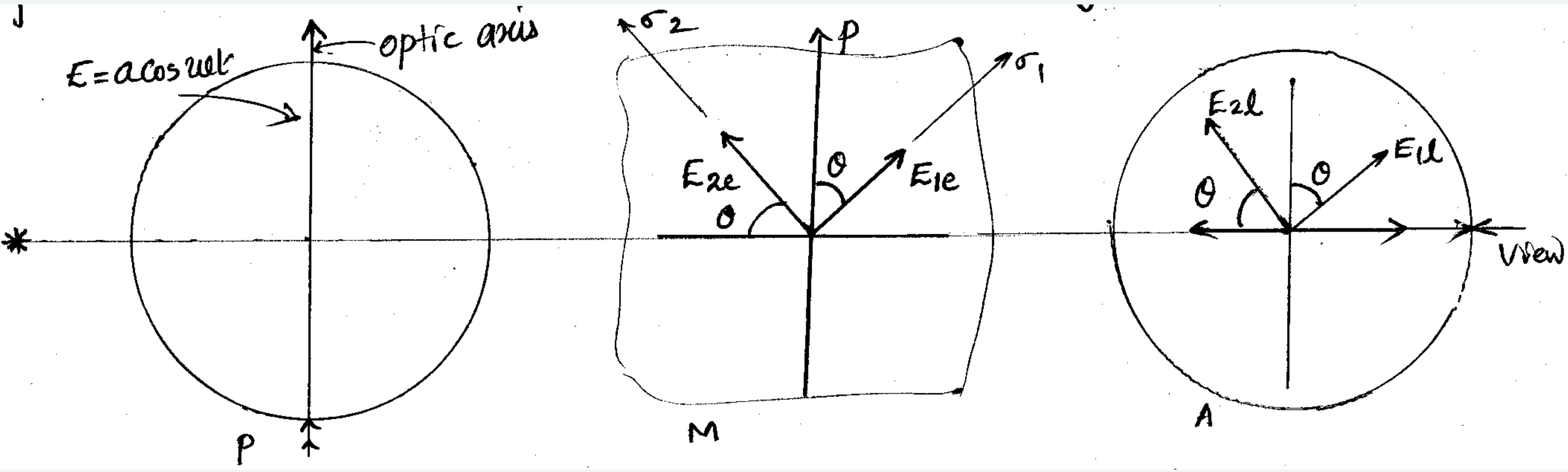
8. **Glass.** It is the first material to be used for photoelastic investigations. It is transparent, free from creep, insensitive to mild temperature changes, isotropic, homogeneous, inexpensive, difficult to machine and has low optical sensitivity.

9. **Gelatine.** The optical sensitivity of gelatine is very high as compared to any other photoelastic material. This is suitable for the investigation of stress problems in which the influence of self-weight is important, *e.g.* in determining the stress distribution in dams and around tunnels. It is highly susceptible to time-edge effect which can be decreased by adding glycerine. Models can be prepared by dissolving gelatine in water and moulding to the desired shape.

10. **Celluloid.** It is a colloidal mixture of nitrocellulose in camphor. It is susceptible to creep and is used for large and complicated models.

POLARISCOPE

Plane Polariscope



POLARISER **MODEL** **ANALYSER**

Plane Polariscope

The plane polarized light beam emerging from the polarizer is

$$E = a \cos \omega t$$

Light vector along the fast axis on entering the model

$$E_{1e} = a \cos \omega t \cos \theta .$$

and along the slow axis

$$E_{2e} = a \cos \omega t \sin \theta .$$

where 'θ' is the angle between maximum principal stress σ_1 and the axis of the polarizer.

Plane Polariscope

Since the light vector E_{1c} travels faster than E_{2c}, on emerging out of the model they develop a phase difference

∴ Light vector leaving along the fast axis of the model

$$E_{1l} = a\cos(\omega t + \Delta)\cos\theta .$$

Along slow axis,

$$E_{2l} = a\cos\omega t \sin\theta .$$

Plane Polariscope

Since the axis of the analyser is oriented at right angles to the polarizer, light vector transmitted through the analyser is,

$$E_t = E_{1l} \sin\theta - E_{2l} \cos\theta.$$

$$= a \cos(\omega t + \Delta) \cos\theta \sin\theta - a \cos\omega t \sin\theta \cos\theta$$

$$= a \sin\theta \cos\theta \left[\cos(\omega t + \Delta) - \cos\omega t\right]$$

$$= \frac{a \sin 2\theta}{2} \times -2 \sin\left(\frac{2\omega t + \Delta}{2}\right) \sin(\Delta/2)$$

$$= -a \sin 2\theta \cdot \sin(\omega t + \Delta/2) \sin(\Delta/2)$$

Plane Polariscope

Intensity of light 'I' is proportional to the square of the amplitude Et,

$$I \propto a^2 \sin^2 2\theta \, \sin^2\left[\omega t + \Delta/2\right] \sin^2(\Delta/2)$$

$$= I_o \sin^2 2\theta \cdot \sin^2\left[\omega t + \Delta/2\right] \sin^2(\Delta/2)$$

I_o = Maximum transmitted light intensity.

Plane Polariscope

Isoclinic Fringe Formation Due to principal Stress Direction (θ

$$I = I_o \sin^2 2\theta \, \sin^2 \frac{\Delta}{2} \, \sin^2\left(\omega t + \frac{\Delta}{2}\right)$$

when $\sin^2 2\theta = 0$, $I = 0$ i.e. the emerging intensity vanishes and gives rise to dark spots in the field of view.

$$\sin^2 2\theta = 0 \quad \text{or} \quad 2\theta = n\pi$$

where $n = 0, 1, 2, 3 \ldots$

Therefore when one of the principal stress direction (θ) coincides with the axis of the polarizer, then the intensity of emergent beam is zero.

Plane Polariscope

The loci of all points where the principal stress direction (σ_1 or σ_2) coincides with the axis of the polarizer forms a fringe pattern called as Isoclinics.

If $\theta = 0^\circ$, It is Called as zero-degree Isoclinic

If $\theta = 30^\circ$, " 30° Isoclinic etc.

Plane Polariscope

Isochromatic Fringe Formation Due to principal stress difference (Δ)

If $\sin^2 \frac{\Delta}{2} = 0$, $I = 0$.

$\Delta = \frac{2\pi d}{\lambda} c (\sigma_1 - \sigma_2)$

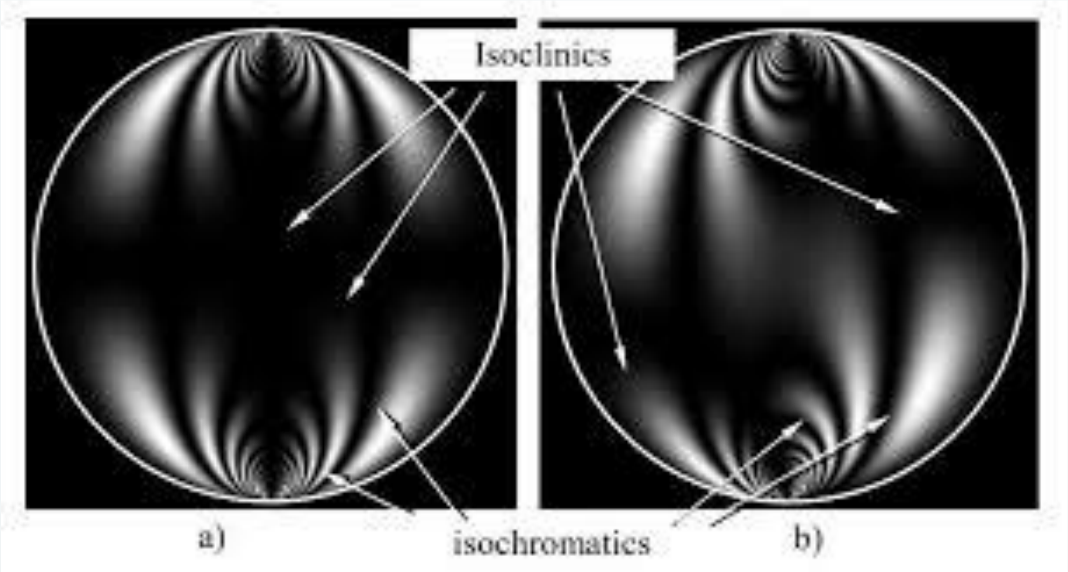
or $\frac{\Delta}{2} = n\pi$

or $\Delta = 2n\pi$, $n = 0, 1, 2, 3 \ldots$

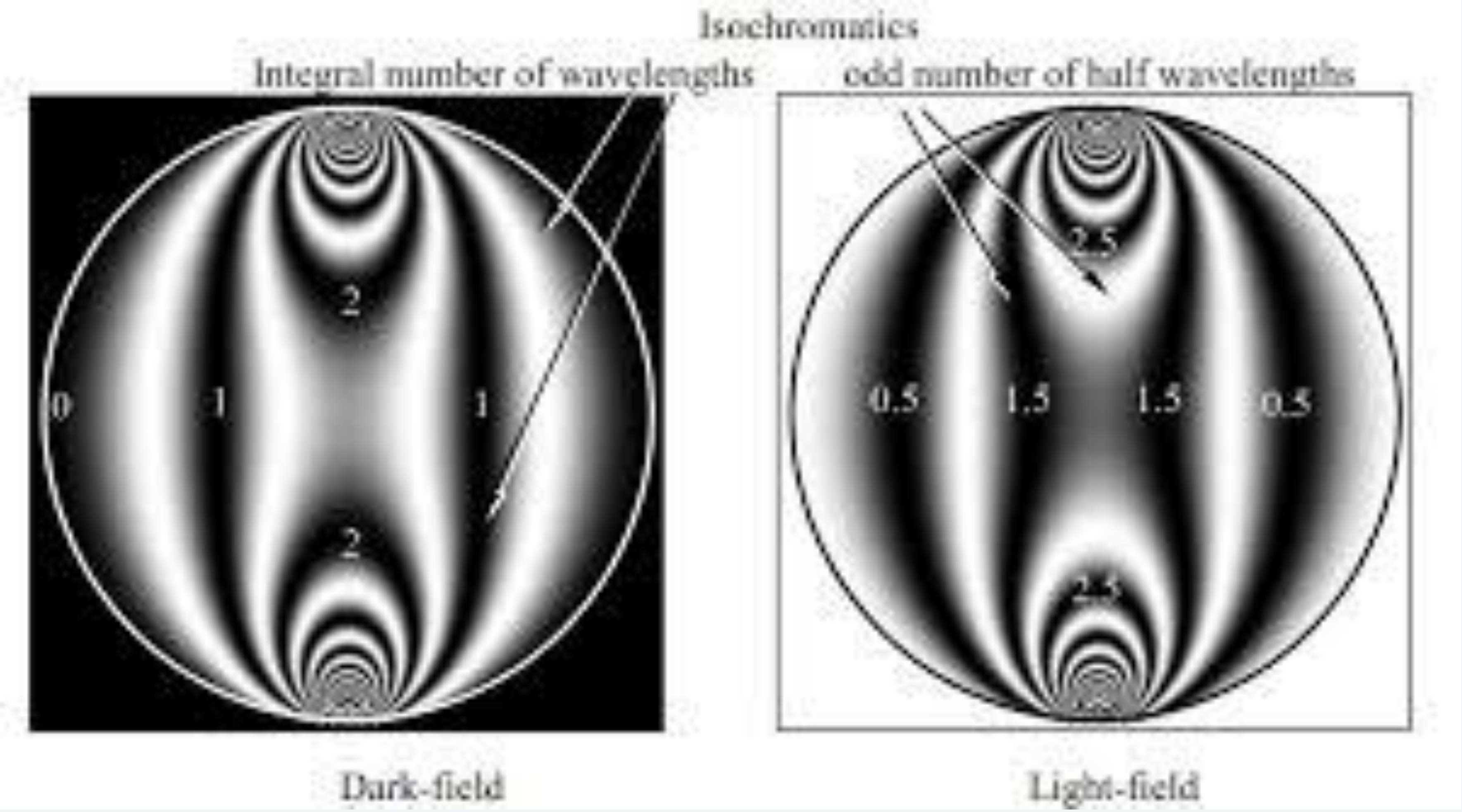
That is when $\Delta = 0$, phase difference is zero. $\therefore (\sigma_1 - \sigma_2) = 0$

Plane Polariscope

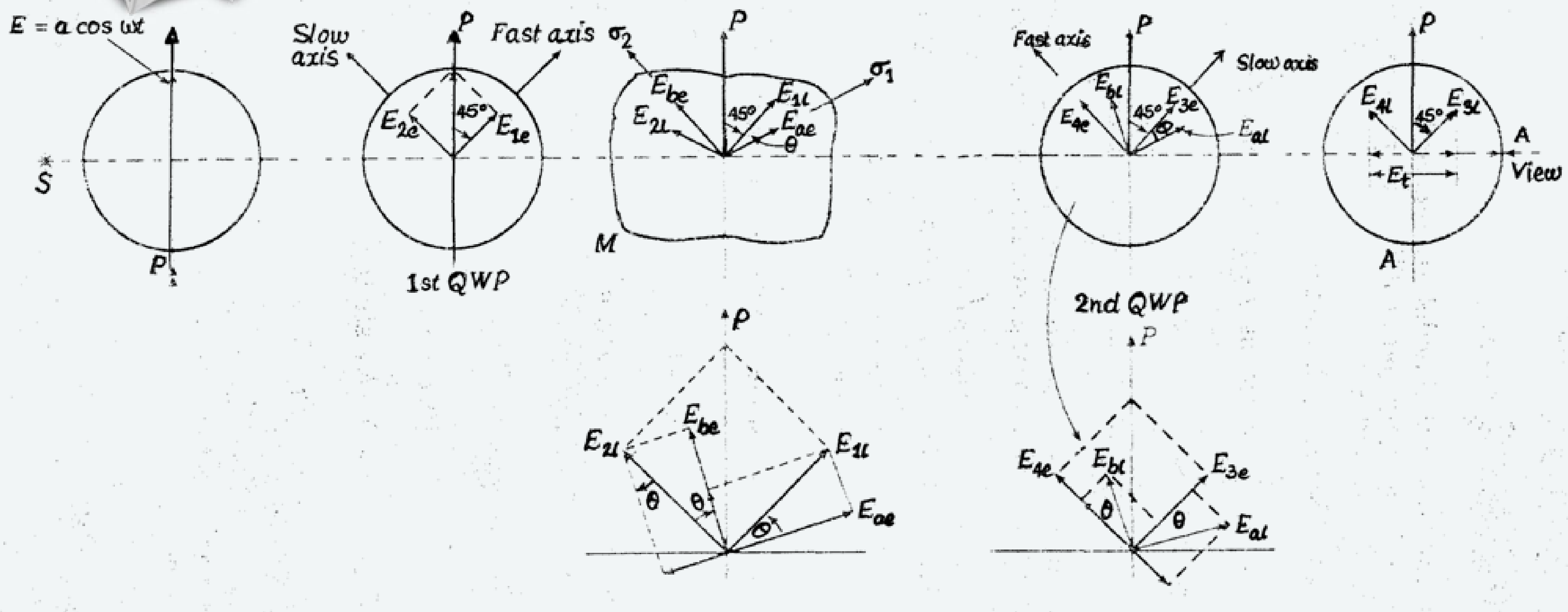
The loci of all points, where the values of $(\sigma_1 - \sigma_2)$ are such that they cause a relative phase difference of $2n\pi$; $n = 0, 1, 2 \ldots$ forms a fringe pattern called as Isochromatics.



Plane Polariscope



Circular Polariscope



Effect of a stressed model in a standard circular polariscope.

E_{1e}, E_{2e} -light vector entering first quarter wave plate

E_{1l}, E_{2l} -light vector leaving first quarter wave plate

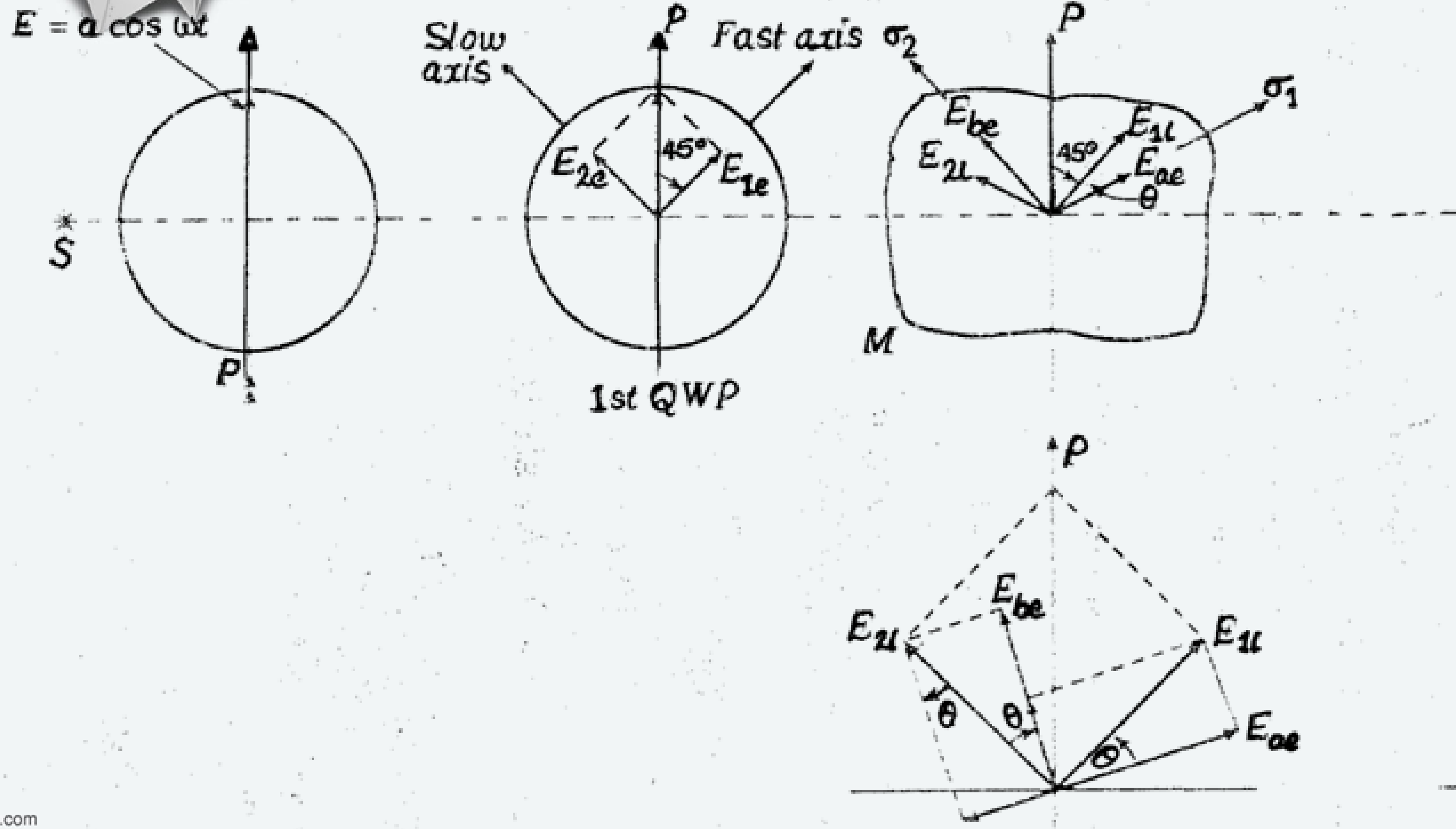
E_{ae}, E_{be} -light vector entering along principal axis of the model

E_{al}, E_{bl} -light vector leaving the model

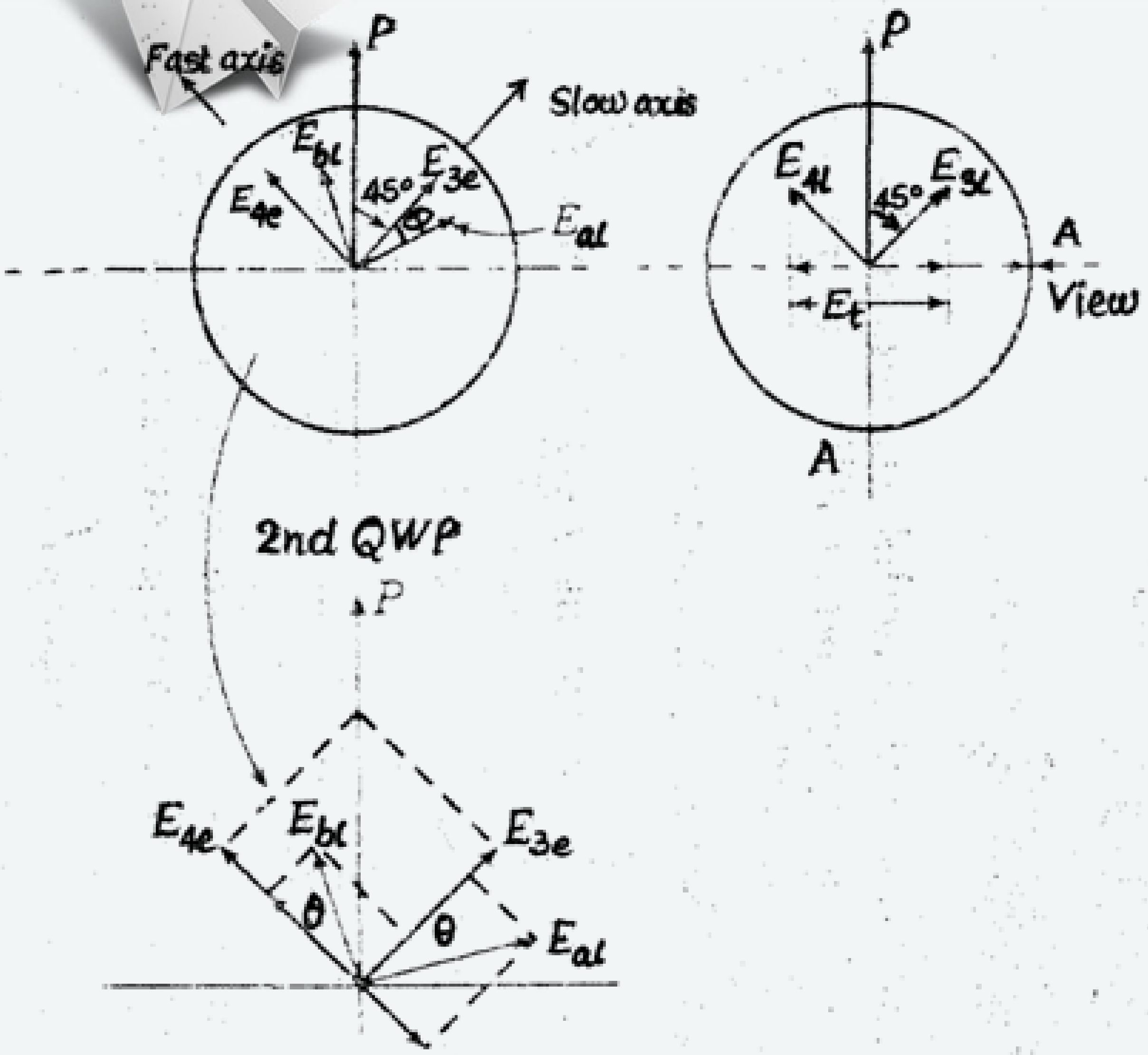
E_{3e}, E_{4e} -light vector *entering* along axis of sec*ond* quarter wave plate

E_{3l}, E_{4l} -light vector leaving the sec*ond* quarter wave plate

Circular Polariscope



Circular Polariscope



Circular Polariscope

E_{1e}, E_{2e} -light vector entering first quarter wave plate

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E_{al}, E_{bl} -light vector leaving the model

E_{3e}, E_{4e} -light vector *entering* along axis of sec*ond* quarter wave plate

E_{3l}, E_{4l} -light vector leaving the sec*ond* quarter wave plate

Circular Polariscope

Effect of a Stressed Model in a Circular Polariscope

(i) Dark-field set up

Consider the Crossed - Crossed Set up of the Circular polariscope.

Light vector leaving the polarizer,

$$E = a \cos \omega t$$

Components of light vector on entering the first QWP become,

$$E_{1e} = a \cos \omega t \cos \pi/4 = a/\sqrt{2} \cos \omega t$$

$$E_{2e} = a \cos \omega t \sin \pi/4 = a/\sqrt{2} \cos \omega t$$

Circular Polariscope

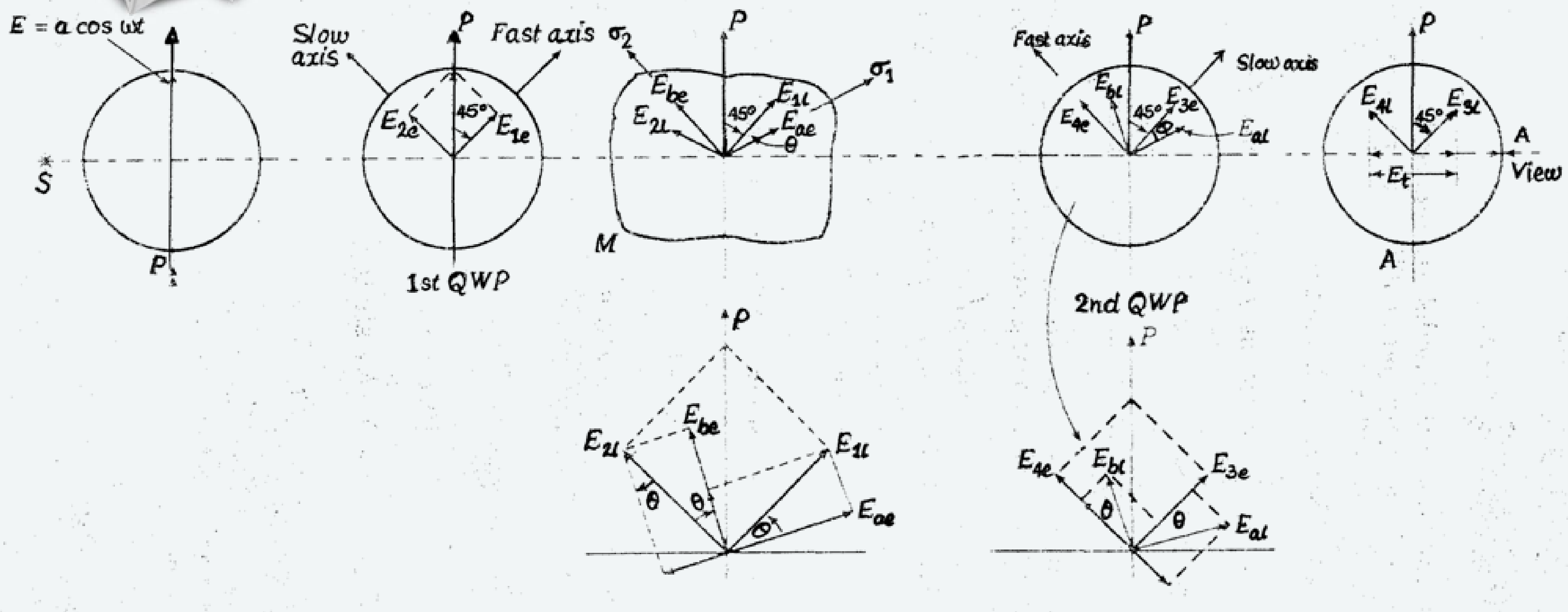
QWP produces a phase difference of $\pi/2$ and converts plane polarized light into circularly polarized light. On leaving the first QWP, the components are

$$E_{1l} = \frac{a}{\sqrt{2}} \cos(\omega t + \pi/2) = -\frac{a}{\sqrt{2}} \sin \omega t$$

$$E_{2l} = E_{2e} = \frac{a}{\sqrt{2}} \cos \omega t$$

If the principal axes of the model are inclined at angle θ with the axis of the QWP, the components of light vector along the principal axis of the model on entering are,

Circular Polariscope



Effect of a stressed model in a standard circular polariscope.

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E_{ae}, E_{be} -light vector entering along principal axis of the model

E_{al}, E_{bl} -light vector leaving the model

E_{3e}, E_{4e} -light vector *entering* along axis of sec*ond* quarter wave plate

E_{3l}, E_{4l} -light vector leaving the sec*ond* quarter wave plate

Circular Polariscope

$$E_{ae} = E_{1l}\cos\theta - E_{2l}\sin\theta$$

$$= -\frac{a}{\sqrt{2}}\sin\omega t\cos\theta - \frac{a}{\sqrt{2}}\cos\omega t\sin\theta$$

$$= -\frac{a}{\sqrt{2}}\left[\sin\omega t\cos\theta + \cos\omega t\sin\theta\right]$$

$$E_{be} = E_{1l}\sin\theta + E_{2l}\cos\theta$$

$$= -\frac{a}{\sqrt{2}}\sin\omega t\sin\theta + \frac{a}{\sqrt{2}}\cos\omega t\cos\theta.$$

The model introduces a phase difference of 'Δ' Components of light vector on leaving the model become

Circular Polariscope

$$E_{al} = -\frac{a}{\sqrt{2}}\left[\sin(\omega t+\Delta)\cos\theta + \cos(\omega t+\Delta)\sin\theta\right]$$

$$= \frac{-a}{\sqrt{2}}\sin(\omega t+\Delta+\theta)$$

$$E_{bl} = \frac{a}{\sqrt{2}}\left[-\sin\omega t\sin\theta + \cos\omega t\cos\theta\right]$$

$$= \frac{a}{\sqrt{2}}\cos(\omega t+\theta)$$

Circular Polariscope

Now the second QWP is crossed to the 1st QWP. The components of the light vector along the axis of 2nd QWP become

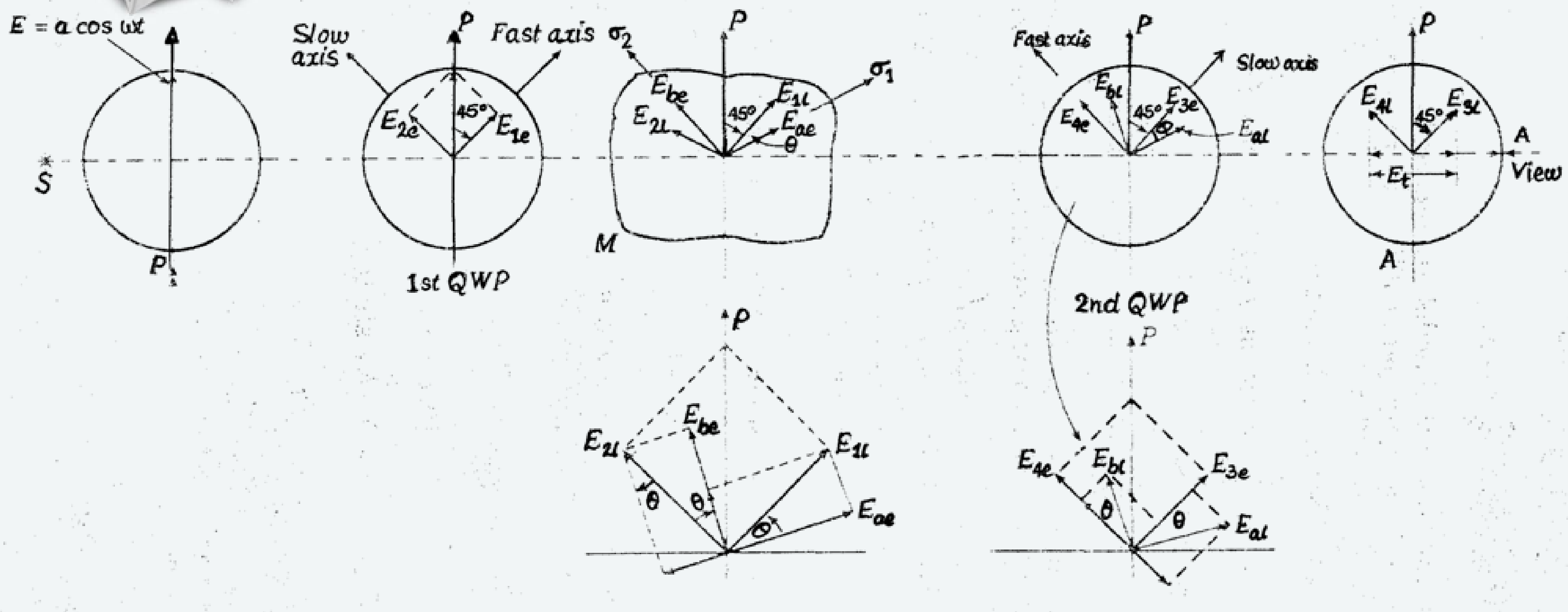
$$E_{3e} = E_{al} \cos\theta + E_{bl} \sin\theta$$

$$= \frac{-a}{\sqrt{2}}\left[\sin(\omega t+\Delta+\theta)\cos\theta - \cos(\omega t+\theta)\sin\theta\right]$$

$$E_{4e} = E_{bl} \cdot \cos\theta - E_{al} \sin\theta$$

$$= \frac{a}{\sqrt{2}}\left[\cos(\omega t+\theta)\cos\theta + \sin(\omega t+\Delta+\theta)\sin\theta\right]$$

Circular Polariscope



. Effect of a stressed model in a standard circular polariscope.

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E_{ae}, E_{be} -light vector entering along principal axis of the model

E_{al}, E_{bl} -light vector leaving the model

E_{3e}, E_{4e} -light vector *entering* along axis of sec*ond* quarter wave plate

E_{3l}, E_{4l} -light vector leaving the sec*ond* quarter wave plate

Circular Polariscope

The second quarterwave plate also produces a phase differen of $\pi/2$. The components of light leaving the second QWP and entering the analyser become

$$E_{3l} = E_{3e}$$

$$E_{4l} = \frac{a}{\sqrt{2}}\left[\cos(\omega t+\theta+\pi/2)\cos\theta + \sin(\omega t+\Delta+\theta+\pi/2)\sin\right.$$

$$= \frac{a}{\sqrt{2}}\left[-\sin(\omega t+\theta)\cos\theta + \cos(\omega t+\Delta+\theta)\sin\theta\right]$$

Circular Polariscope

The resultant light vector transmitted through the Crossed analyser become

$$E_t = E_{3l} \cos \pi/4 - E_{4l} \cos \pi/4$$

$$= \frac{1}{\sqrt{2}} (E_{3l} - E_{4l})$$

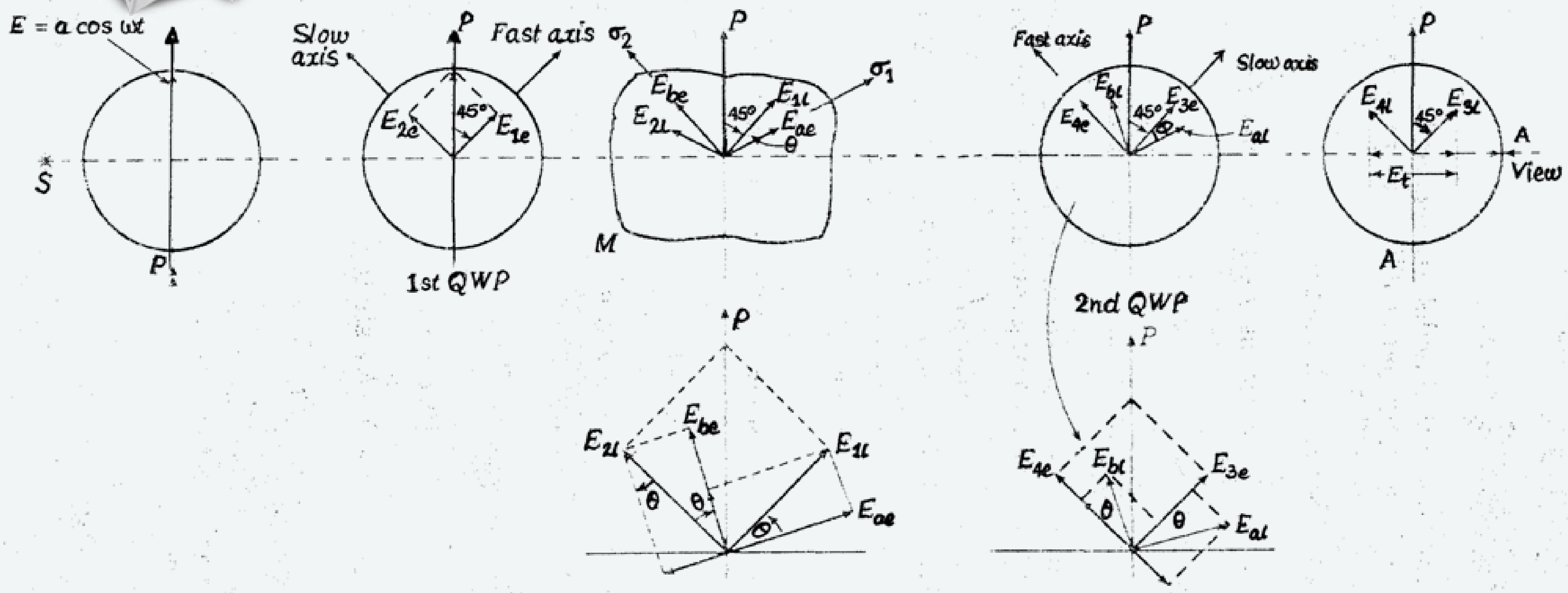
$$= \frac{a}{2} [\cos(\omega t + \theta) \sin \theta - \sin(\omega t + \Delta + \theta) \cos \theta$$

$$+ \sin(\omega t + \theta) \cos \theta - \cos(\omega t + \Delta + \theta) \sin \theta]$$

$$= \frac{a}{2} [\sin(\omega t + 2\theta) - \sin(\omega t + \Delta + 2\theta)]$$

$$= -a \cos(\omega t + 2\theta + \Delta/2) \sin \Delta/2$$

Circular Polariscope



Effect of a stressed model in a standard circular polariscope.

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E_{ae}, E_{be} -light vector entering along principal axis of the model

E_{al}, E_{bl} -light vector leaving the model

E_{3e}, E_{4e} -light vector *entering* along axis of sec*ond* quarter wave plate

E_{3l}, E_{4l} -light vector leaving the sec*ond* quarter wave plate

Circular Polariscope

Intensity of light $I \propto E_t^2$

$$\therefore \quad I \propto a^2 \cos^2\left(\omega t + 2\theta + \Delta/2\right) \sin^2 \Delta/2$$

$$I = I_o\, a^2 \cos^2\left(\omega t + 2\theta + \Delta/2\right) \sin^2 \Delta/2$$

$I = 0$ when

(a) Effect of frequency.

when $(\omega t + 2\theta + \Delta/2) = (2n+1)\pi/2$

then $\cos^2(\omega t + 2\theta + \Delta/2) = 0$, $n = 0, 1, 2 \ldots$

Hence $I = 0$.

Circular Polariscope

But the frequency ω is very high and any extinction produced by it cannot be detected by eye or any other photographic equipment. Hence the isoclinics are automatically eliminated. Thus

$$I = I_0 \sin^2 \Delta/2$$

Circular Polariscope

(b) Effect of stress difference.

when $\Delta/2 = n\pi, n = 0, 1, 2 \ldots$

$$\sin^2 \Delta/2 = 0.$$

$$\therefore I = 0$$

This type of extinction is identical to that of plane polariscope and referred to as isochromatic fringe pattern.

MODULE 5

Calibration of Photoelastic Materials

Calibration of Photoelastic Materials

The photoelastic materials has to be calibrated to determine the material fringe value 'f_σ' so as to convert the fringe orders into stress.

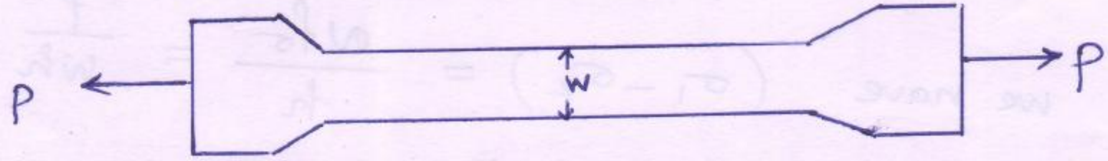
$$\sigma_1 - \sigma_2 = \frac{N f_\sigma}{h}$$

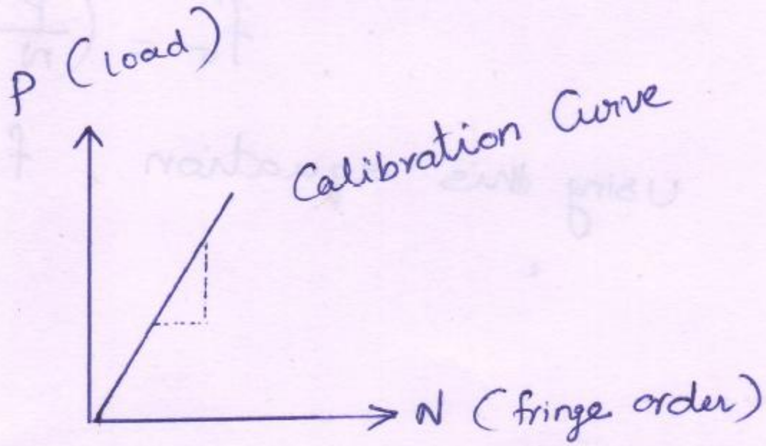
$$f_\sigma = \frac{h}{N}(\sigma_1 - \sigma_2)$$

The following methods may be used to calibrate a photoelastic material.

Calibration of Photoelastic Materials

1. Simple Tensile Specimen





Prepare a simple tensile specimen with width 'w' and thickness 'h' as shown.

Calibration of Photoelastic Materials

Under load 'P', the uniform stress in the specimen is $(\sigma_1 - \sigma_2) = \frac{P}{wh}$,

The member is in a plane state of stress with $\sigma_1 = \frac{P}{wh}$, $\sigma_2 = 0$.

Calibration of Photoelastic Materials

The tension specimen can be viewed in a Circular polariscope and the formation of successive integral number of fringe orders as the load 'P' is continuously varied, can be observed. A graph is plotted between load 'P' and fringe order 'N' and the slope (P/N) is determined.

Calibration of Photoelastic Materials

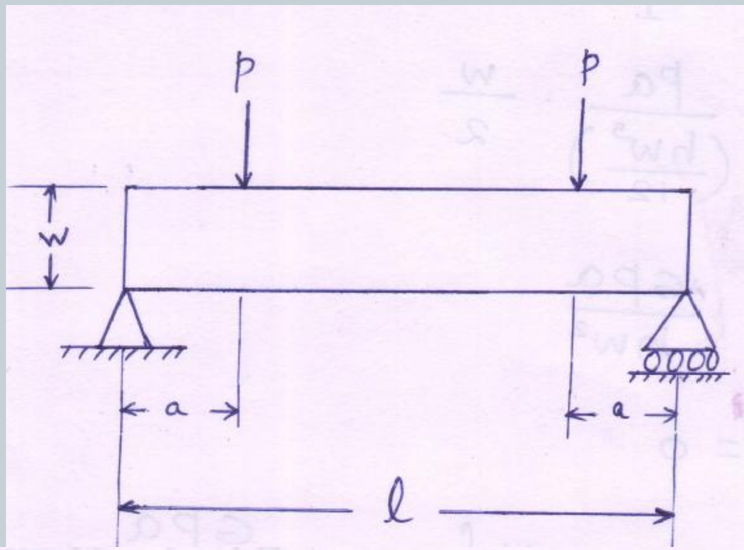
we have $(\sigma_1 - \sigma_2) = \frac{N f_\sigma}{h} = \frac{P}{wh}$

$$\therefore \quad f_\sigma = \left(\frac{P}{N}\right)\frac{1}{w}$$

Using this equation, f_σ can be Calculated.

Calibration of Photoelastic Materials

2. Beam Under Pure Bending



pure bending in the beam may be produced by applying equal loads P at a distance 'a' from the ends of the beam of length 'l' as shown.

Calibration of Photoelastic Materials

A rectangular beam of thickness 'h' and width 'w' may be used and subjected to pure bending to determine f_σ.

The uniform bending moment 'M' at the middle portion of the beam is

$$M = Pa.$$

Calibration of Photoelastic Materials

The stress in the beam,

$$\sigma_1 = \frac{My}{I}$$

$$= \frac{Pa}{\left(\frac{hw^3}{12}\right)} \cdot \frac{w}{2}$$

$$= \frac{6Pa}{hw^2}$$

$$\sigma_2 = 0$$

Calibration of Photoelastic Materials

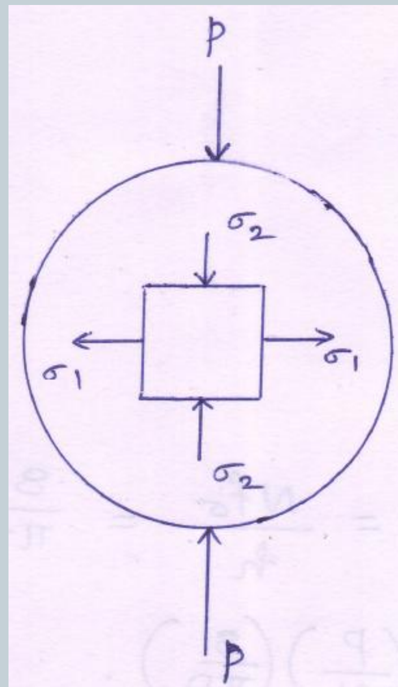
$$\therefore \sigma_1 - \sigma_2 = \frac{N f_\sigma}{h} = \frac{6Pa}{hw^2}$$

$$f_\sigma = \left(\frac{P}{N}\right)\left(\frac{6a}{w^2}\right)$$

From the graph plotted between P and N, the slope of the curve (P/N) can be substituted in above equation to determine 'f_σ'.

Calibration of Photoelastic Materials

3. Circular Disc Under Diametral Compression



A Circular disc under diametral Compression is also frequently used as a calibration specimen.

Calibration of Photoelastic Materials

A Circular disc under diametral Compression is also frequently used as a calibration specimen. If 'D' is the diameter of the disc and 'h' is the thickness, the stress distribution along the horizontal diameter is given by,

$$\sigma_x = \sigma_1 = \frac{2P}{\pi h D}\left[\frac{D^2 - 4x^2}{D^2 + 4x^2}\right]^2$$

$$\sigma_y = \sigma_2 = \frac{-2P}{\pi h D}\left[\frac{4D^4}{(D^2 + 4x^2)^2} - 1\right]$$

Where 'x' is the distance measured along the diameter from the Centre of the disc.

Calibration of Photoelastic Materials

At the Centre of the disc, $x = 0$

$$\sigma_1 = \frac{2P}{\pi h D}$$

$$\sigma_2 = \frac{-6P}{\pi h D}$$

$$\sigma_1 - \sigma_2 = \frac{8P}{\pi h D}$$

Calibration of Photoelastic Materials

we have $\sigma_1 - \sigma_2 = \dfrac{N f_\sigma}{h} = \dfrac{8P}{\pi h D}$

$$\therefore \quad f_\sigma = \left(\frac{P}{N}\right)\left(\frac{8}{\pi D}\right).$$

From the P/N curve, slope can be obtained and substituted to calculate f_σ.

MODULE 5

Compensation Techniques

Compensation Techniques

The method of Compensation Consists in bringing the existing fringe order to an integral Value. It is a technique in which partial modification of relative retardation either by addition or subtraction is brought about so that the fractional fringe order at a point becomes integral. Then by knowing the amount of relative retardation added or subtracted, the actual fringe order at that point can be ascertained.

Compensation Techniques

Three methods are commonly employed in practice. In the first method, a known or measurable amount of retardation is either added or subtracted to make the final retardation value an integral value. This is done by putting a crystal combination (called Babinet - Soleil Compensator) in front of the model and suitably adjusting the value of retardation given by the crystal combination.

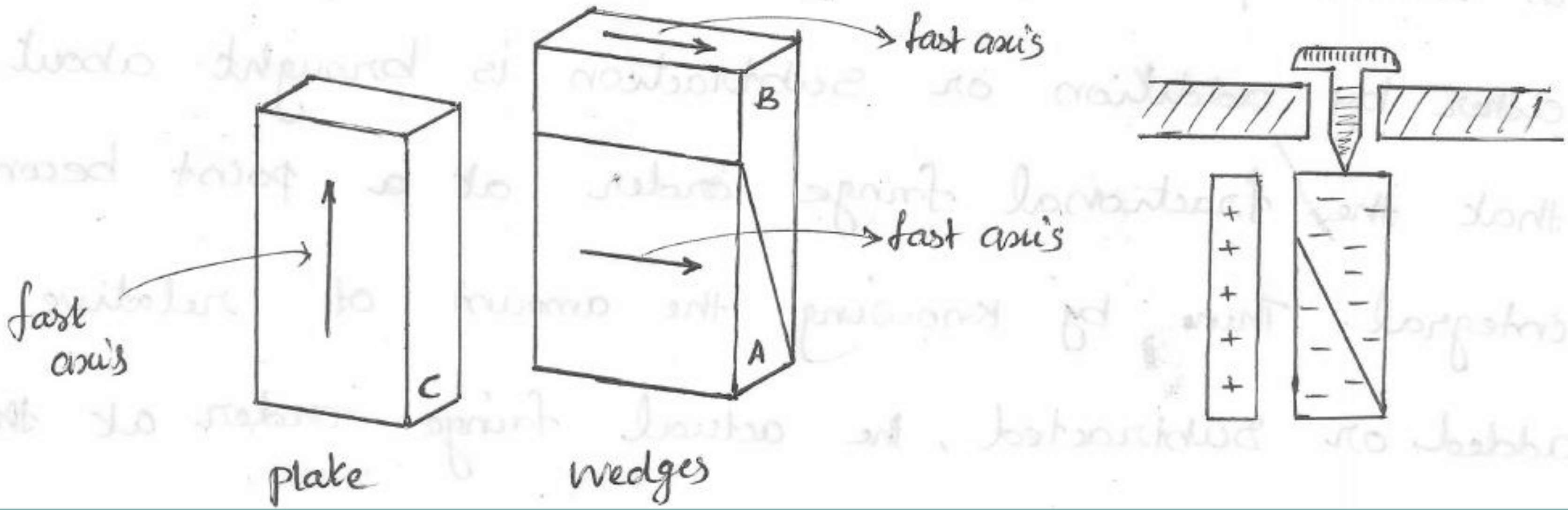
Compensation Techniques

In the second method, a quarterwave plate is used to reduce the ellipse of light Coming out of the model into a linearly polarized light and determining the orientation of this by means of the analyser. This is Known as Tardys method of Compensation. A variation of this, Known as Friedel's method of Compensation is also discussed.

Compensation Techniques

Babinet – Soleil Compensator

It consist of a quartz plate of uniform thickness and two quartz wedges. The optical axes of the plate and the wedges are mutually orthogonal.



Compensation Techniques

A and B are two wedges with their fast axes similarly oriented, so that the two wedges together form one rectangular piece of uniform thickness over a limitted portion. By moving one wedge w.r.t the other, the thickness of the Combination can be varied. Next to wedge Combination is a quartz plate C.

Compensation Techniques

The retardation given by plate C can be Cancelled partially or fully by varying the thickness of the wedge Combination. Thereby the Compensator Can add or subtract relative retardations within a given range. The micrometer screw is calibrated in number of wavelengths of retardation added or subtracted along a marked axis of the Compensator.

Compensation Techniques

In practice, a point is selected on the model where the fringe order is to be established precisely. Then the compensator is kept before or after the model and is oriented along the principal stress axis at the point of interest in the model. From the zero position, micrometer head is turned either one way or the other until a dark fringe passes through the point. observation will generally indicate whether the higher order fringe or lower order fringe has moved to the point and thus indicating whether the integral value has been obtained by addition or subtraction of retardation given by the compensator.

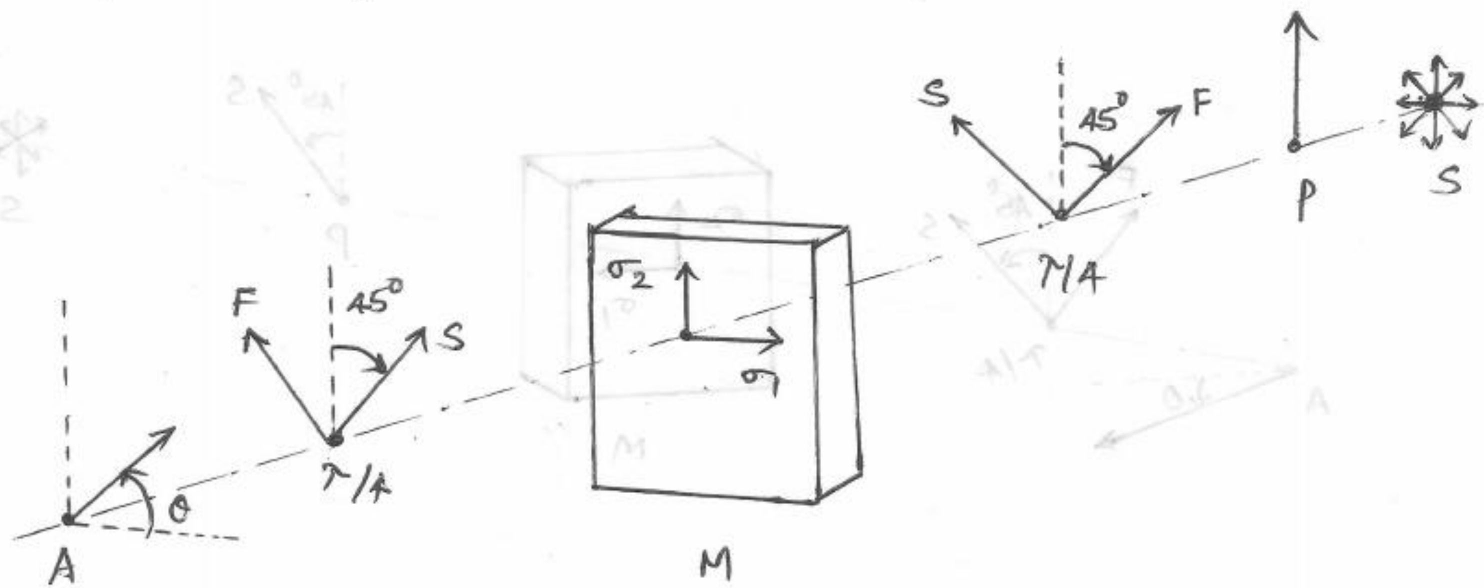
Compensation Techniques

Tardy's Method of Compensation

In this method, no auxiliary equipment is required and the analyser of the polariscope serves as the compensator. In this, the polarizer is aligned with the direction of the principal stress σ_1 at the point of interest and all other elements of the polariscope are rotated relative to the polarizer so that a standard dark field set up exists. Then the analyser alone is rotated to obtain compensation.

Compensation Techniques

The compensation can be achieved in the following way.



Compensation Techniques

i) Using the plane polariscope set up, determine the principal stress directions at the point of interest by rotating the crossed polarizer and analyzer until an isoclinic passes through the point.

ii) Now rotate only the quarter-wave plates of a circular polariscope to obtain a standard dark-field arrangement.

iii) Rotate only the analyzer then until an isochromatic fringe coincides with the point. Determine the angle 'γ' that the analyzer has rotated.

Compensation Techniques

iv) If the n^{th} order fringe moves to the point, the fringe order at the point is

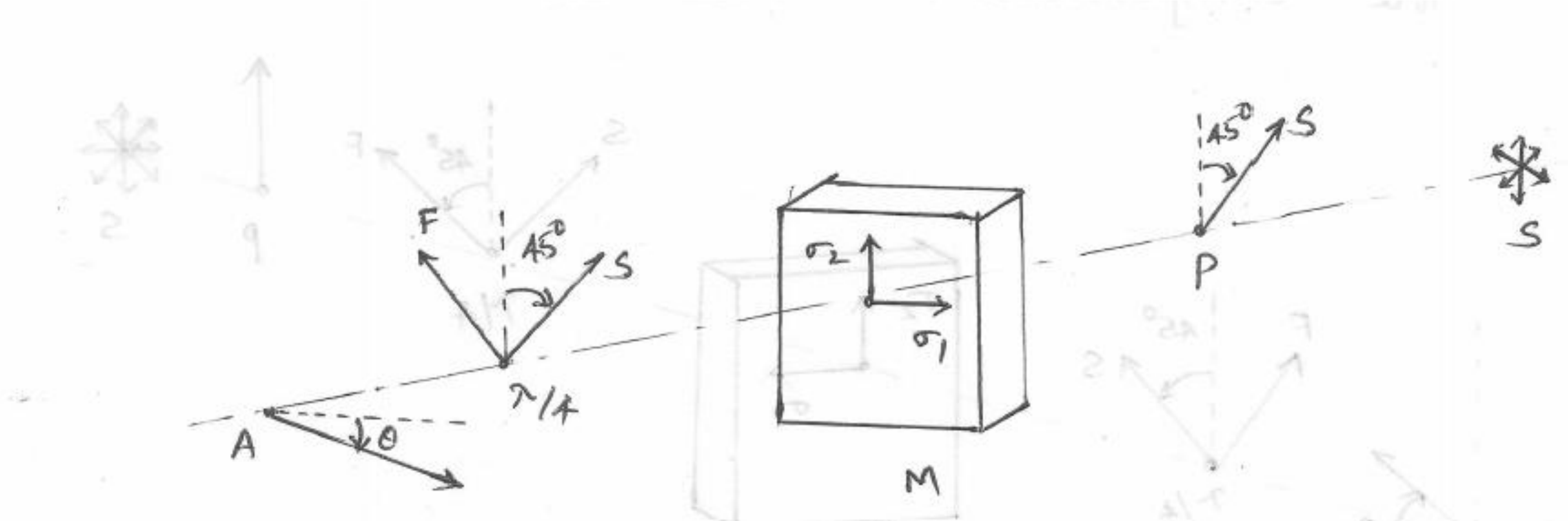
$$N = n + \frac{\gamma}{\pi}$$

If the $(n+1)^{st}$ order fringe moves to the point as the analyzer rotates in the opposite direction, then

$$N = (n+1) - \frac{\gamma}{\pi}$$

Compensation Techniques

Friedel's Method of Compensation
(Senarmont Method of Compensation)



This is similar to Tardy's method without the first quarter-wave plate.

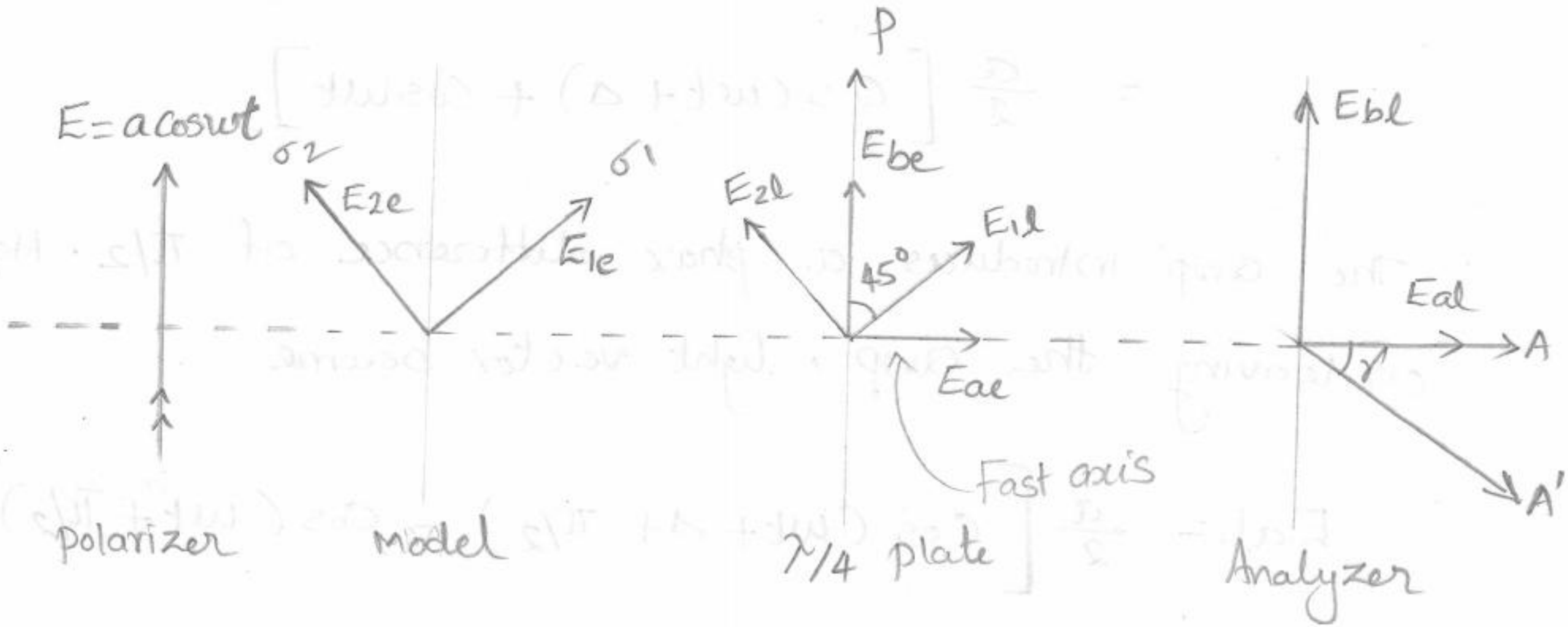
Compensation Techniques

The following steps are involved for this method.

1. Remove the first quarter wave plate
2. Rotate the system of polarizer and analyzer so that their axes make angles of 45° with the principal directions in the model at the point of interest.
3. Rotate second quarter wave plate until one axis is parallel to the axis of the polarizer.

Compensation Techniques

4. Rotate the analyzer until the integral fringe order is obtained at the point of interest.



Compensation Techniques

Light vector from the polarizer is

$$E = a \cos \omega t$$

Light vector on entering the model are

$$E_{1e} = a \cos \omega t \cos 45^\circ = \frac{a}{\sqrt{2}} \cos \omega t$$

$$E_{2e} = a \cos \omega t \sin 45^\circ = \frac{a}{\sqrt{2}} \cos \omega t$$

Compensation Techniques

The model introduces a phase difference of 'Δ'. Therefore on leaving the model, the light components are

$$E_{1L} = \frac{a}{\sqrt{2}} \cos(\omega t + \Delta)$$

$$E_{2L} = \frac{a}{\sqrt{2}} \cos \omega t$$

The fast axis of the 1st QWP is set to 90° to the polarizer axis. Hence on entering the QWP, the light components become

Compensation Techniques

$$E_{ae} = E_{1l} \cos 45^\circ - E_{2l} \cos 45^\circ$$

$$= \frac{a}{2}\left[\cos(\omega t + \Delta) - \cos \omega t\right]$$

$$E_{be} = E_{1l} \cos 45^\circ + E_{2l} \cos 45^\circ$$

$$= \frac{a}{2}\left[\cos(\omega t + \Delta) + \cos \omega t\right]$$

The QWP introduces a phase difference of $\pi/2$. Hence on leaving the QWP, light vector become

Compensation Techniques

$$E_{al} = \frac{a}{2}\left[\cos\left(\omega t + \Delta + \pi/2\right) - \cos\left(\omega t + \pi/2\right)\right]$$

$$= \frac{a}{2}\left[-\sin(\omega t + \Delta) + \sin \omega t\right]$$

$$E_{bl} = E_{be} = \frac{a}{2}\left[\cos(\omega t + \Delta) + \cos \omega t\right]$$

Now the analyzer is rotated through an angle γ to obtain integral fringe order at the point of interest.

Compensation Techniques

Light transmitted through the analyzer is

$$E_t = E_{al}\cos\gamma - E_{bl}\sin\gamma$$

$$= -\frac{a}{2}\left\{\left[\sin(\omega t+\Delta) - \sin\omega t\right]\cos\gamma + \left[\cos(\omega t+\Delta) + \cos\omega t\right]\sin\gamma\right\}$$

$$= -\frac{a}{2}\left[2\cos\left(\omega t+\frac{\Delta}{2}\right)\sin\frac{\Delta}{2}\cos\gamma + 2\cos\left(\omega t+\frac{\Delta}{2}\right)\cos\frac{\Delta}{2}\sin\gamma\right] = 0$$

Compensation Techniques

Hence $\sin\frac{\Delta}{2}\cos\gamma' + \cos\frac{\Delta}{2}\sin\gamma' = 0$

$$\sin\left(\frac{\Delta}{2}+\gamma'\right) = 0$$

$$\frac{\Delta}{2}+\gamma' = n\pi, \quad n = 0, 1, 2 \dots$$

$$\frac{\Delta}{2} = n\pi - \gamma'$$

$$\boxed{N = \frac{\Delta}{2\pi} = n - \frac{\gamma'}{\pi}}$$

MODULE 5

Seperation Techniques

Seperation Techniques

Separation techniques are used to determine individual values of $\sigma 1$ and $\sigma 2$.

1. Use of lateral extensometer
2. At free boundary
3. Oblique incidence method

Seperation Techniques

1. Use of lateral extensometer

From Hooke's law, for the plane state of stresses,

$$\epsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$

$$\epsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$

$$\epsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y)$$

Seperation Techniques

If 'h' is the original thickness of the model before being stressed and $h + \Delta h$ is the thickness after loading,

then

$$\epsilon_z = \frac{\Delta h}{h}$$

$$\sigma_x + \sigma_y = -\frac{E}{\nu}\epsilon_z$$

$$= -\frac{E}{\nu} \cdot \frac{\Delta h}{h}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 + \sigma_2 = \sigma_x + \sigma_y$$

Seperation Techniques

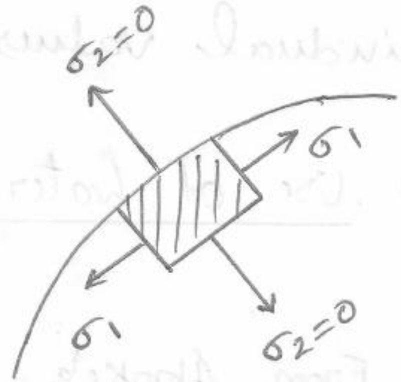
The instrument used to determine the change in thickness, i.e. the change in the lateral dimension of the model, is called lateral extensometer.

Since $(\sigma_1 + \sigma_2) = \sigma_x + \sigma_y$ and $(\sigma_1 - \sigma_2)$ are known, the individual values of σ_1 and σ_2 can be determined.

Seperation Techniques

2. At Free Boundary

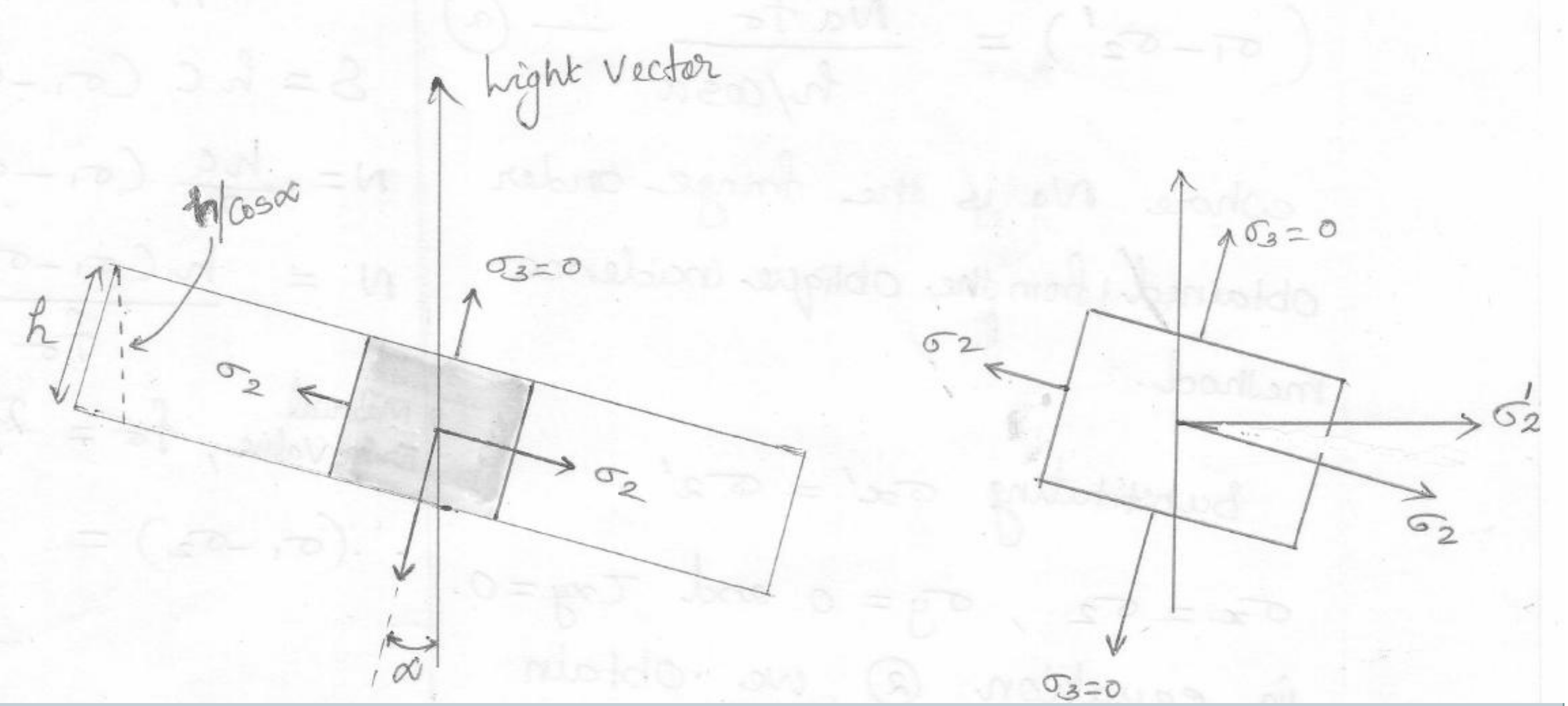
when a boundary of the model is not loaded directly, it is called a free boundary. The normal and shear stresses on a plane tangential to a free boundary is therefore zero.



The principal stress axes are normal and tangential to the boundary. One of the principal stress is zero. ($\sigma_2=0$)

Hence isochromatics near a free boundary give the values of the non-vanishing principal stress σ_1.

3. Oblique incidence method



Seperation Techniques

The stress-optic law was obtained under the condition that light was passing through the stressed model at normal incidence. If the stressed model is rotated through an angle 'α' as shown, an oblique incidence isochromatic fringe pattern can be obtained in a circular polariscope. This fringe pattern provides additional data, which can be used to seperate the principal stresses σ_1 & σ_2.

Seperation Techniques

Consider the model is rotated about σ_1 axis clockwise by an angle 'α'. Let the thickness of the model be 'h' so that light traverses a distance of $h/\cos\alpha$ through the oblique model. Now the fringe pattern developed in a circular polariscope is related to the principal stresses σ_1 and σ_2'.

Seperation Techniques

We have, from stress optic law

$$N = \frac{hc}{\lambda}(\sigma_1 - \sigma_2)$$

$$N = \frac{h(\sigma_1 - \sigma_2)}{f_\sigma}$$

$$\therefore (\sigma_1 - \sigma_2) = \frac{N f_\sigma}{h} \quad \text{--- } (1)$$

Seperation Techniques

Now in the Case of oblique incidence, the equation ① become

$$(\sigma_1 - \sigma_2') = \frac{N_a f_\sigma}{h/\cos\alpha} \quad \text{— ⓐ}$$

where N_a is the fringe order obtained from the oblique incidence method.

$$\sigma_2' = \sigma_2 \cos^2\alpha$$

Seperation Techniques

∴ eqn ⓐ becomes

$$\left(\sigma_1 - \sigma_2 \cos^2 \alpha\right) = \frac{N_a f_\sigma}{h/\cos\alpha} \quad \text{--- (2)}$$

By solving equations ① and ②, it is possible to seperate principal stresses.

Moire Method of Strain Analysis

Moire Method

The word "moire" derives its origin from a silk fabric which when super-posed on itself exhibits patterns of light and dark bands.

MOIRE PHENOMENON

Two arrays of alternately placed transparent and opaque lines or dots when moved relative to each other, result in fringe patterns consisting of alternately placed bright and dark bands which are termed **moire fringes**.

Moire Method

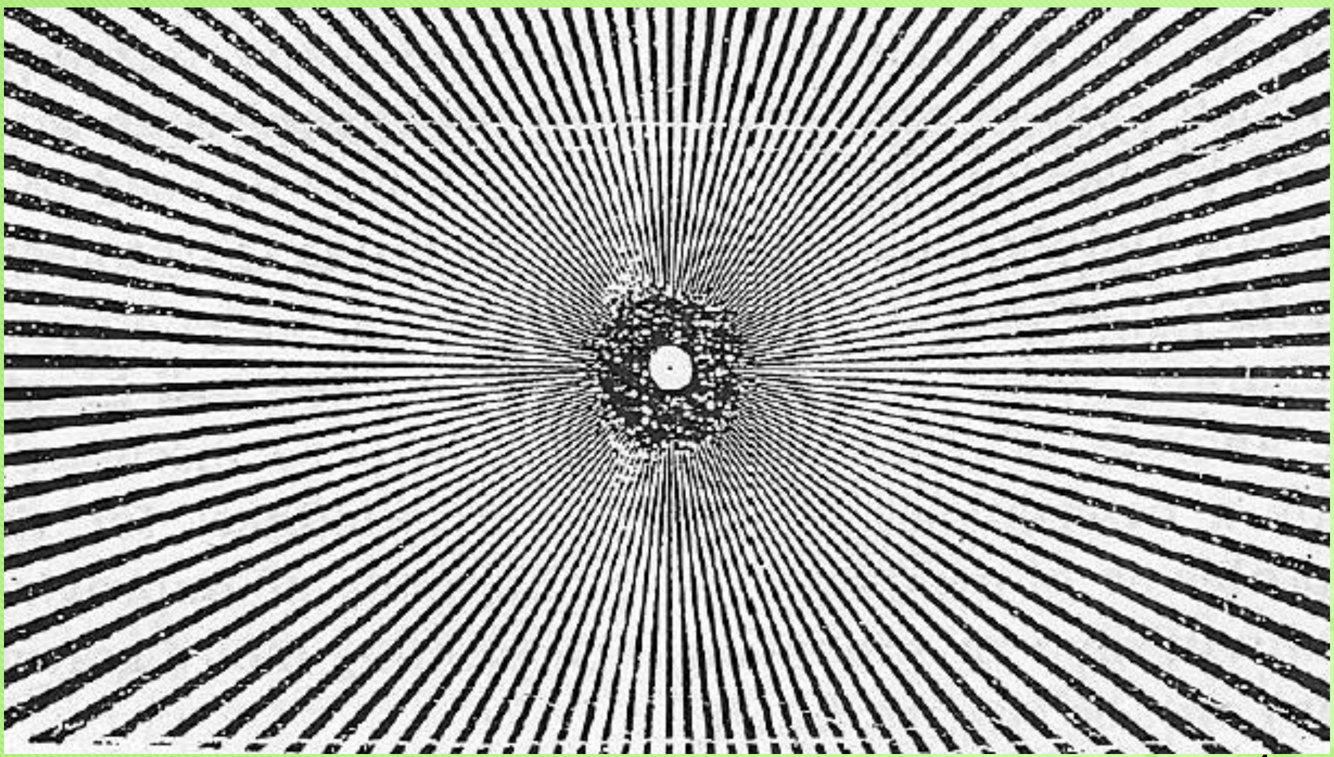
- An ensemble of equispaced opaque lines separated by transparent slits or lines, which are used to obtain moire fringes is called a **grating**.

Parallel Line Grating

Moire Method

- In a radial line grating opaque and transparent lines are alternate radial lines.

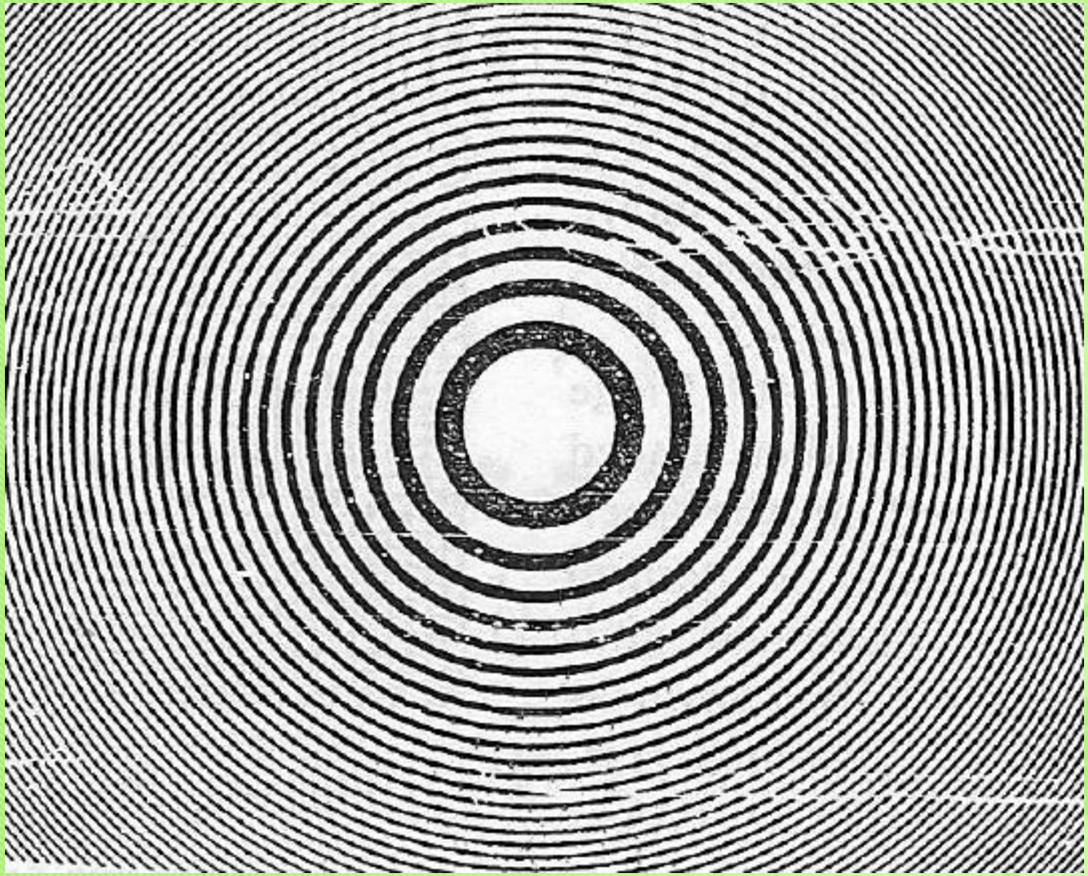
Radial Line Grating



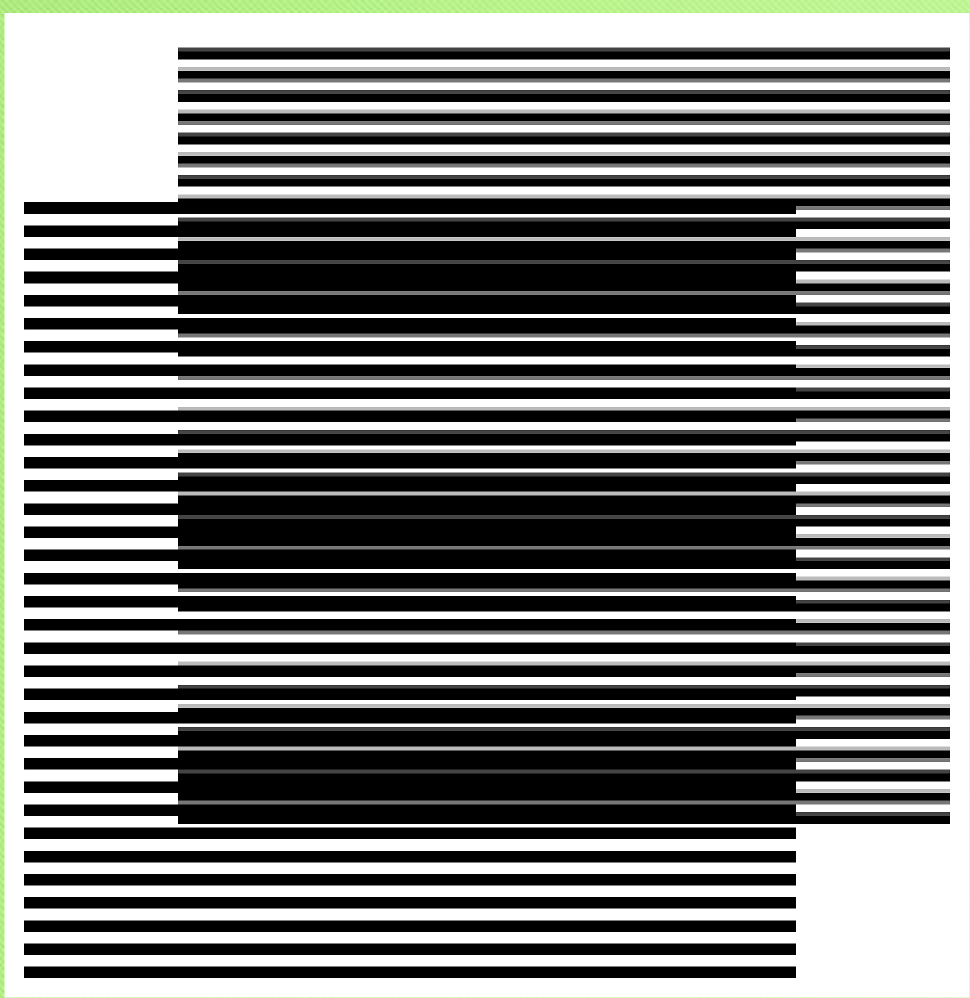
Moire Method

- In a circular line grating opaque and transparent lines are circles of varying radii.

Circular Line Grating

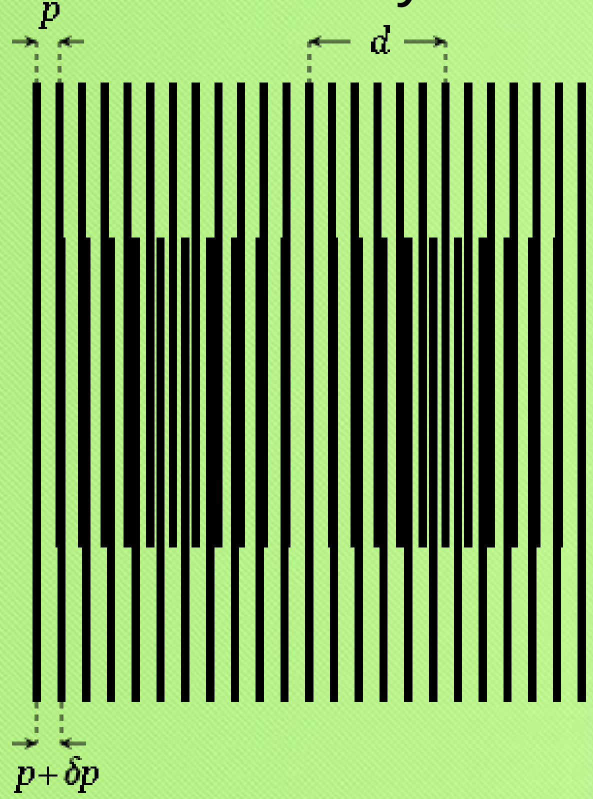


Moire Method



Moire Method

- The distance between corresponding points in a grating is called the **pitch** and is denoted *by ‘p’.*
- The **density** of the grating, which represents the number of lines per unit length is denoted by ‘d’.

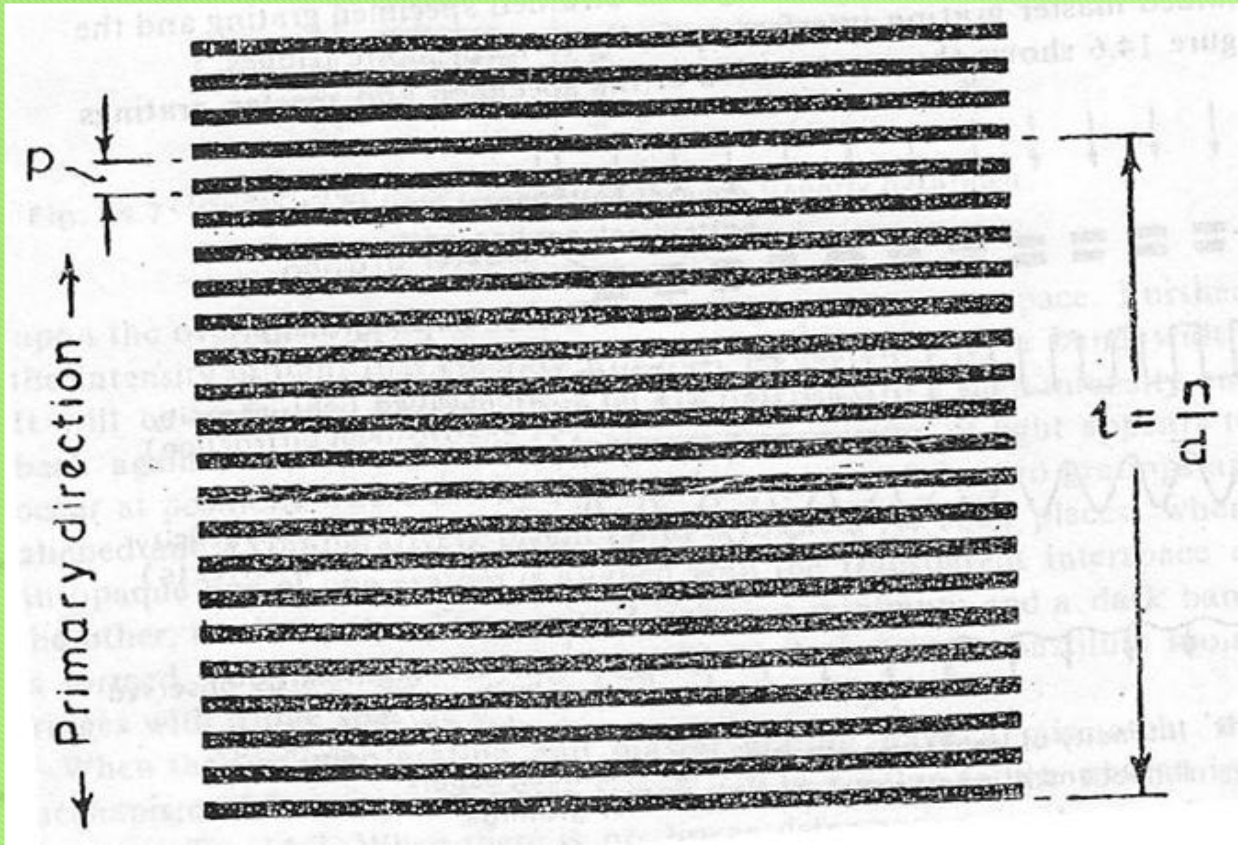


Moire Method

- The direction perpendicular to the lines in the plane of grating is called the **primary direction**, while the direction parallel to the lines is called the **secondary direction.**

- When two gratings of the same category having a mismatch of pitches (i.e. with different pitches and therefore densities) are placed one above the other, moire fringes are formed even without relative movement between the two.

Moire Method



Moire Method

- Of the two gratings used for fringe formation, one grating is either bonded to, or etched or printed on the specimen being analyzed and is termed as “**model grating**" or "**specimen grating**".

- The other grating is known as "**master grating**" or "**reference grating**".

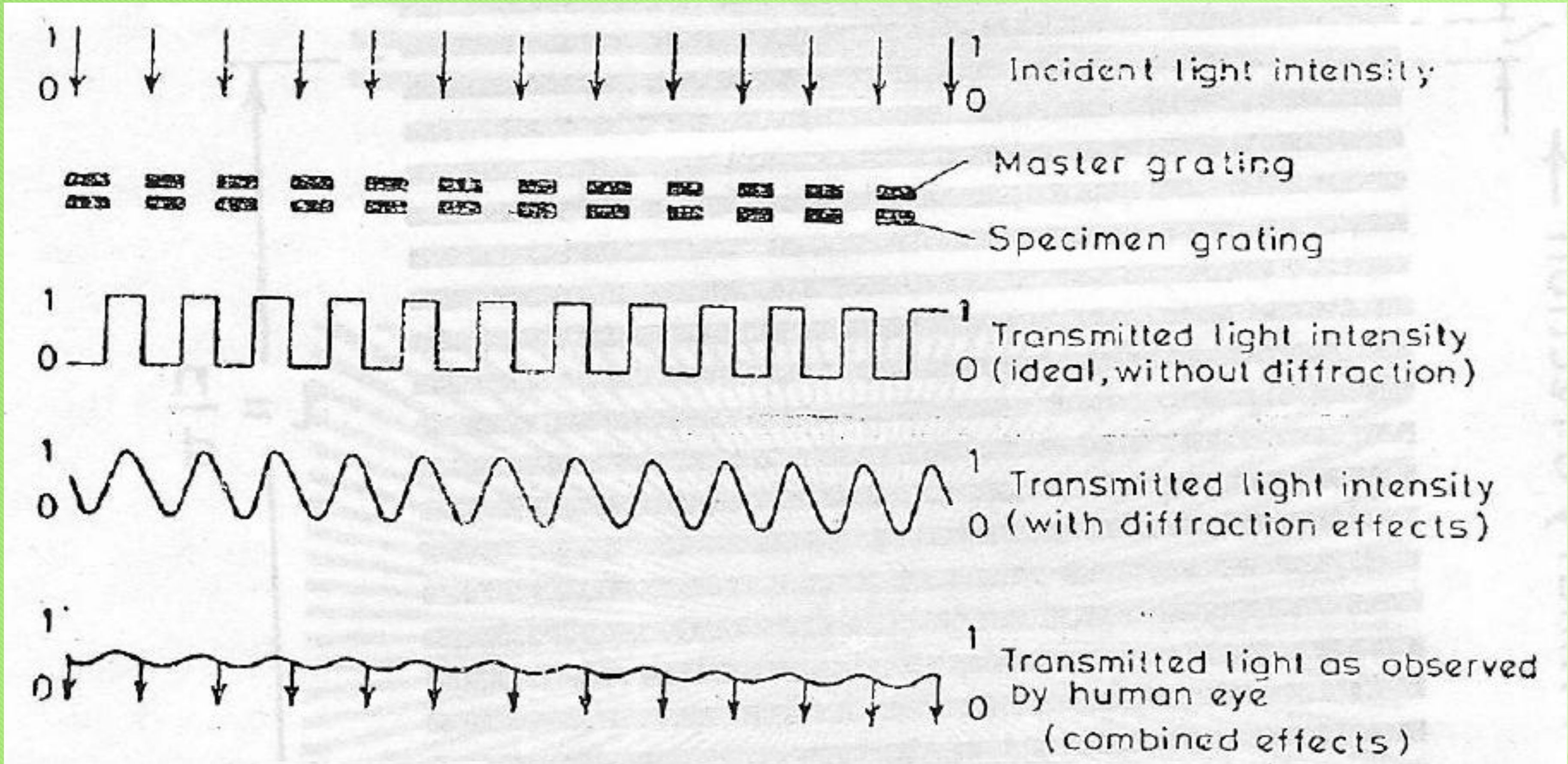
Moire Method

- The specimen grating undergoes deformation depending on the state of strain on the surface and is accompanied by a change of spacing (i.e. pitch) between the lines of grating.

- The master grating, which is not strained, does not undergo any change in the spacing between lines of grating.

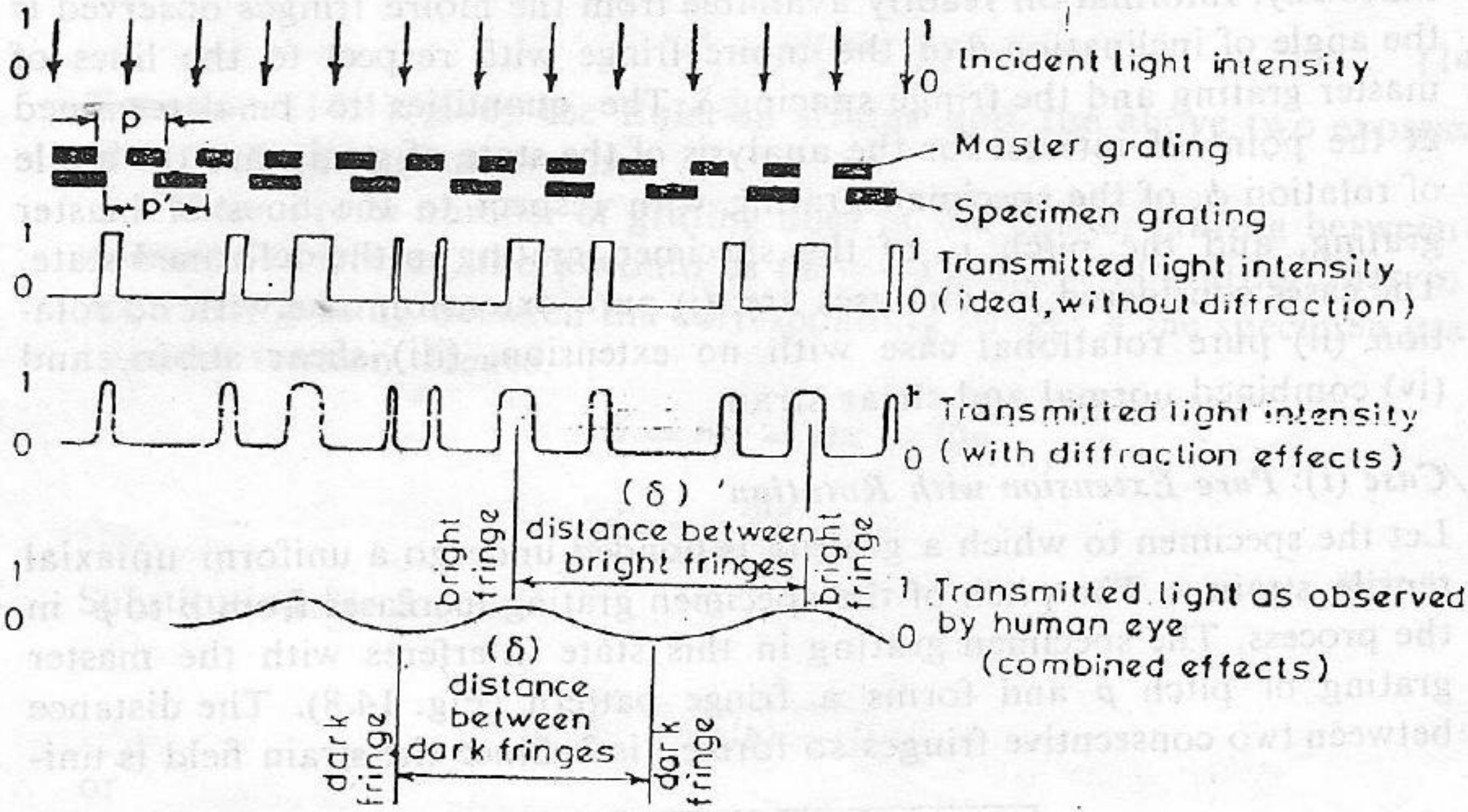
- The strained specimen grating and the unstrained master grating interfere optically to form moire fringes.

Moire Method



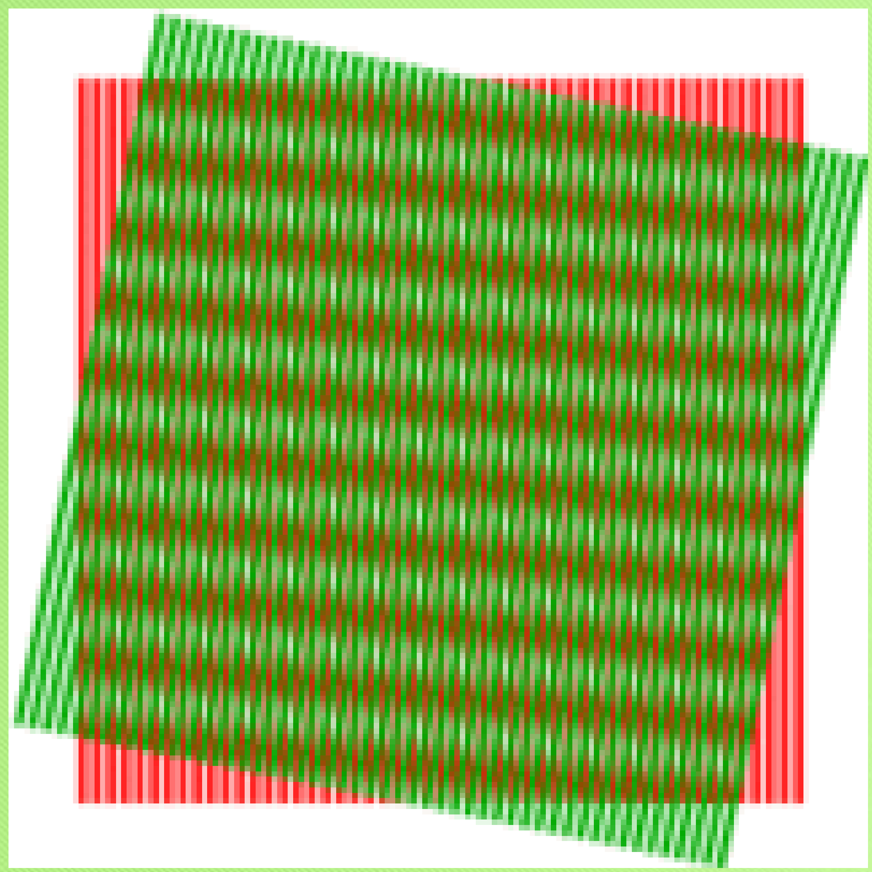
Intensity of light transmitted through identical, superposed and aligned specimen and master gratings

Moire Method



Intensity of light transmitted through linearly deformed specimen grating and master grating

Moire Method



Effect of changing angle.